EOC Math I Remediation Lessons

Nevada Department of Education

The Candy Machine: Using Explicit and/or Recursive Functions

Enduring Understanding

(Do not tell students; they must discover it for themselves.)

Represent real world contexts using explicit and/or recursive functions. Reason quantitatively, using arithmetic sequences.

Launch

A grocery store has a candy machine. Each time a person inserts a quarter, 7 candies come out of the machine. The machine holds 15 pounds of candy. Each pound of candy contains about 180 individual candies.

1. Write a function that represents the number of candies in the machine for any given number of customers. About how many customers will there be before the machine is empty?
2. About how many candies are in the machine after 47 customers?
3. Represent the amount of money in the machine for any given number of customers.

4. To avoid theft, the store owners take all the money out when they think the machine has about $25 in it. The owners cannot tell how much money is in the machine without opening it up, so they choose when to remove the money by how many candies are left in the machine. About how full should the machine look when they reach $25? How do you know?

Understand the Problem

- Are there any words you don't understand?
- What are you asked to find?
- Is there enough information to find a solution?
- Can you restate the problem in your own words?
- Or, what information do you need to find?

Develop a Plan

- There are many reasonable ways to solve a problem. With practice students will build skill in choosing efficient strategies.
- Do not validate/invalidate any strategies, but ensure that students have a place to start (even if you know it will not work).
- Do not force your plan/reasoning on students.

Investigate

- Let students engage in a productive struggle.
- Monitor as students work.
- Do not offer feedback.
- Only ask questions.
    - Why did you choose that number?
    - What assumptions did you make?
    - Explain what you are doing here.
    - What does that solution mean?

Questions for Students as they Work

(If you observe _____, then you might ask _____.)

If students have a hard time getting started, then ask:

- How might you organize your data in order to observe a pattern?
- What totals can you calculate?
- Do you think you have enough data points? How could you find more?

If students have 15-7*x*, then ask:

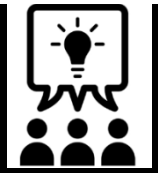
- What does 15 represent in the problem situation?
- What does 7 represent?

If students have a table of values, but no equations, then ask:

- What patterns do you notice in your table? How might you describe that pattern in symbols?
- How might your table help you know if your equation is correct?

If students have a difficult time determining how full the machine looks when containing $25, then ask:

- If you know how many candies have been removed, how might that help you determine about how full the machine looks?
- Could drawing a picture help?
- What does "how full" mean to you?



Debrief

Whole or Small Group Discussion

- Debriefing formats may differ (e.g., whole-class discussion, small-group discussion). It will be beneficial for students to view student work as a gallery walk or similar format.
- Have students sequence multiple representations in an order that moves from less to more mathematical sophistication.
- Allow students to question each other and explain their choices, using mathematical reasoning. If students struggle, model your own questioning strategies.
- Encourage students to notice similarities, differences, and generalizations across strategies.
- Provide purposeful feedback and ask questions.

Sample Solutions

(If you observe _____, then you might ask _____.)

If students have exact values to describe how full the machine is when containing $25, then ask:

- Which makes more sense in this context; exact or approximate? Why?

If students have functions written in different forms (explicit or recursive), then ask:

- Which makes more sense in this context?
- Which is more useful in this situation?
- When might one form be more useful than another?
- Can we write both types (recursive, explicit) for the same context?
- What similarities or differences are present in the functions?
- Are any of the functions equivalent? How do you know?

If students leave units out of their answers, then ask:

- What are the quantities associated with your variables?
- Have you included *domain*, *range*, *continuity*, and other key features of functions in the discussion?
- How does the domain effect continuity?
- What type of sequence is modeled (arithmetic, geometric)
- What type of function is modeled (linear, exponential)

If students' samples are in one notation, then ask:

- Could you introduce other notations?

Synthesize and Apply

When students have independently arrived at the Enduring Understanding, engage them in solving these extension problems. Monitor student work and facilitate discussions by asking questions.

Extension Problem 1

Miss Math has a gumball machine in her classroom. Each time a person inserts a quarter, 3 gumballs come out of the machine. The machine holds approximately 650 gumballs.

- Write an explicit function to find the amount of gumballs in the machine after *n* customers.
- Write an explicit function to find the amount of money in the machine after *n* customers.
- About how many customers will there be before the machine is empty?

Miss Math wants to buy some supplies for her students using the gumball money. She needs $30 to purchase the supplies.

- About how full will the gumball machine look when there is enough money inside?

Extension Problem 2

Joshua has $320 in his savings account at the end of May. He plans to deposit $50 per month into his account.

- Write an explicit function that represents the amount of money in Joshua's savings account for any given number of months.
- Write a recursive function that represents the amount of money in Joshua's savings account for any given number of months.
- How much money is in Joshua's account in June, July, and August of the same year?
- Assuming Josh doesn't make any withdraws and continues to deposit $50 per month, how much money will Joshua have 2 years from May?
- If Joshua keeps saving at this rate, how long will it take him to save $2000?

Extension Problem 3

Juanita wants to save $8,000 to buy a used car. She currently has $500 in their account and plan to save $50 per month. The function $f(0) = 500,\ f(1) = 500 + f(n-1)$ models the recursive situation.

- How long will it take Juanita to save $8,000 to purchase the car?
- If Juanita wants to buy the car in 3 years, how much money does she need to save each month?

References

Common Core State Standards Initiative. (2010). *Common core state standards for mathematics.* Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.

Standards

This task might address the following standards (standards might vary based on discussion):

- HSA.CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- HAS.REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- HSF.IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, If the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

- HSF.IF.C.8.a Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Polya, G. (2014). How to solve it: *A new aspect of mathematical method.* Princeton, NJ: Princeton University Press.

Name_____

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