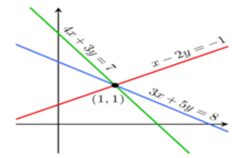


Sandia Aerial Tram

Enduring Understanding

(Do not tell students; they must discover it for themselves.)

Students will construct a linear equation given a table of input-output values. Students will utilize the function to predict a value beyond the table of values.

Standards

HSF.LE.A.2 Students will develop further understanding of how to construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).

HSF.LE.A.1.b Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

Launch

Introduce the Task

Albuquerque boasts one of the longest aerial trams in the world. The tram transports people up to Sandia Peak. The table shows the elevation of the tram at various times during a particular ride.

Minutes into the Ride	2	5	9	14
Elevation in Feet	7069	7834	8854	10,129

a. Write an equation for a function (linear, quadratic, or exponential) that models the relationship between the elevation of the tram and the number of minutes into the ride. Justify your choice.

b. What was the elevation of the tram at the beginning of the ride?

c. If the ride took 15 minutes, what was the elevation of the tram at the end of the ride?

Understand the Problem

- Are there any word(s) you don't understand?
- What is the question or task asking you to answer?
- Is there enough information to find a solution?
- Restate the problem in your own words.
- What additional information do you need to find?

Develop a Plan

- There are many reasonable ways to solve a problem. With practice, students will build the necessary skills to choose an efficient strategy for the given problem.
- Ensure that students have a place to start and that the task/problem has the ability to be scaffolded.
- Caution should be exercised to not force your plan/reasoning on students.

Investigate

Productive Struggle

- Let students engage in productive struggle.
- Monitor as students work.
- Offer positive constructive feedback.
- Ask questions such as…
    - Why did you choose that number?
    - What assumptions did you make?
    - Explain what you are doing here.
    - What does that solution mean?

Questions for Individuals as they Work

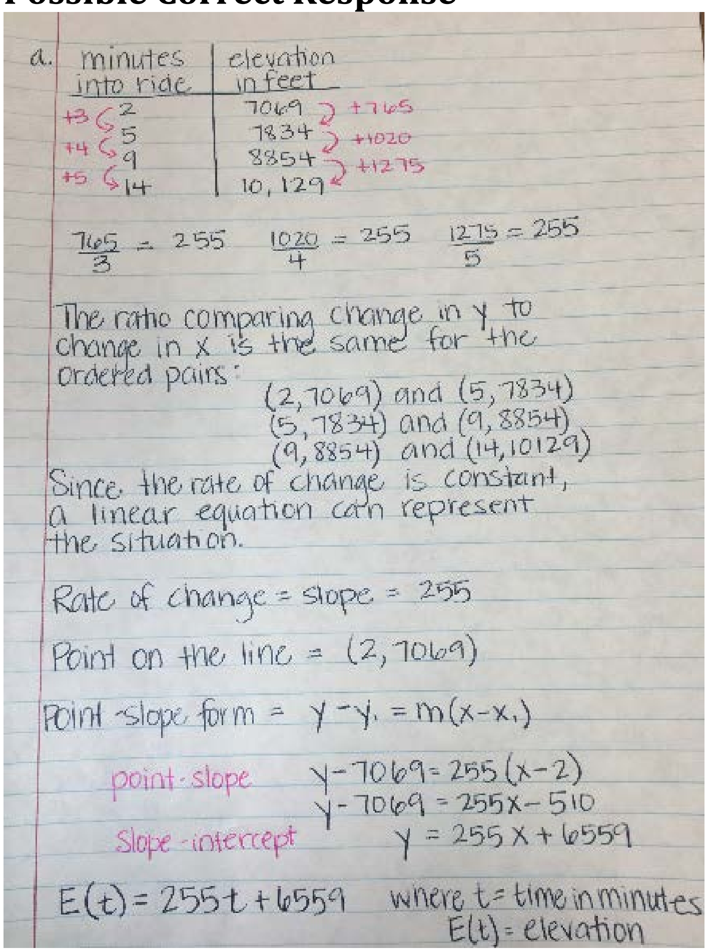
Students are unable to start the problem… What do we know? What do we need to know? How can you tell if the function represents a linear, quadratic, or exponential function? What have you tried? What other methods can you try (graph, for example)?

The student is unable to determine the function equation…What is the difference between linear, quadratic, and exponential functions? Can you describe the graphs of linear, quadratic, and exponential functions? Can you state the parent function or equation for a linear, quadratic, and exponential function? What is the independent variable? What is the dependent variable? Have you found the rate of change? Is the rate of change constant?

The student is unable to create an equation…What kind of equation do you need to write? What do you need to know? What quantities are being compared? What is the relationship between those quantities? What are the different forms of a linear equation? How do you find slope? If you know slope, what else could you use to write an equation? (e.g., if the student knows slope, m, and a point, (x, y) → use point-slope form: $y - y_1 = m(x - x_1)$; if the student knows slope, m, and y-intercept, b → use slope-intercept form**:** $y = mx + b$). How can you find the y-intercept?
The student is unable to find slope…What is slope? What does slope describe? What is the slope formula? How can you find slope using two points/from a table/from a graph?
The student is unable to find the y-intercept …What is the definition of y-intercept? What do you know about the ordered pair of a y-intercept? How many minutes have elapsed at the beginning of a ride? If x = 0, what is y? Can you expand your table by examining patterns?
The student is finished…What did you do to verify your solution? Is your solution reasonable in the context of the problem? How do you know? What patterns did you find?

Sample Solutions

Possible Correct Response



a.

minutes into ride	elevation in feet
2	7069
5	7834
9	8854
14	10, 129

+3, +4, +5 (minutes); +765, +1020, +1275 (elevation)

$\frac{765}{3} = 255$ $\frac{1020}{4} = 255$ $\frac{1275}{5} = 255$

The ratio comparing change in y to change in x is the same for the ordered pairs:
(2, 7069) and (5, 7834)
(5, 7834) and (9, 8854)
(9, 8854) and (14, 10129)

Since the rate of change is constant, a linear equation can represent the situation.

Rate of change = slope = 255

Point on the line = (2, 7069)

Point-slope form = $y - y_1 = m(x - x_1)$

point-slope $y - 7069 = 255(x - 2)$
$y - 7069 = 255x - 510$
Slope-intercept $y = 255x + 6559$

$E(t) = 255t + 6559$ where t = time in minutes
E(t) = elevation

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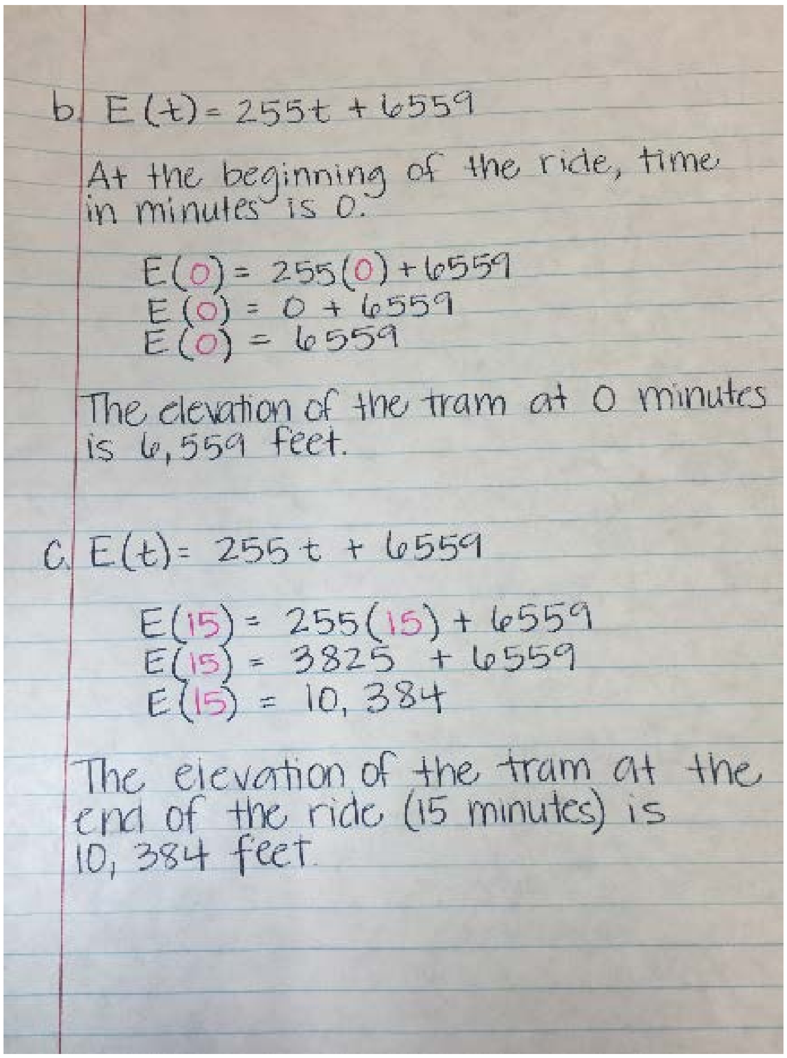
Part a: How did you get this answer? Why did you calculate differences between entries on the table? Is the table easier to read/interpret/understand in horizontal form (as it was given) or in vertical form (as shown in the solution)? What does the ratio comparing change in y to change in x tell you in the context of this question? Why isn't it a quadratic function? Why isn't it an exponential function? What are the different forms of linear equations? What is the dependent variable? What is the independent variable? What do the variables mean in this situation? Which form of a linear equation is easiest to use with the information you have been given? Explain the process you used to transform the equation from point-slope form to slope-intercept form. How should it be written in function form?



b. $E(t) = 255t + 6559$

At the beginning of the ride, time in minutes is 0.

$E(0) = 255(0) + 6559$
$E(0) = 0 + 6559$
$E(0) = 6559$

The elevation of the tram at 0 minutes is 6,559 feet.

c. $E(t) = 255t + 6559$

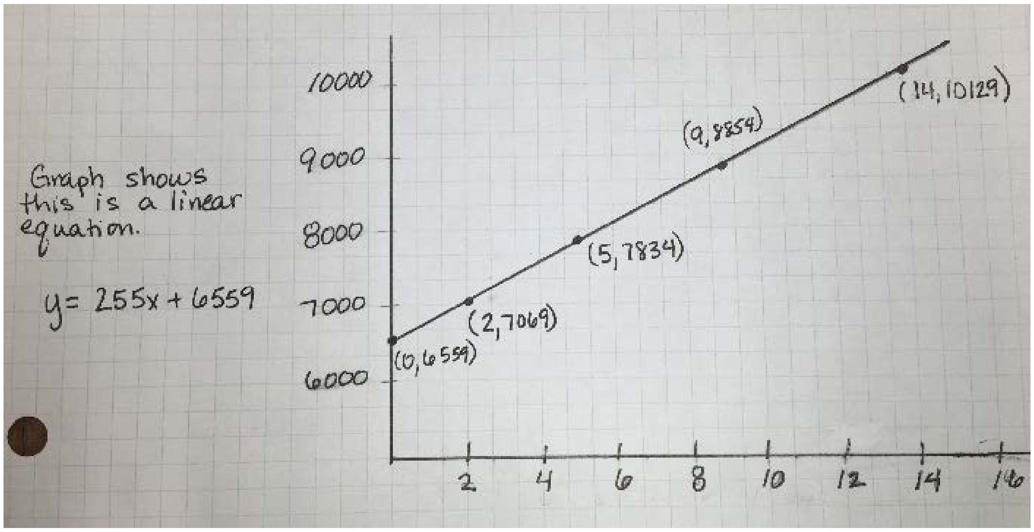
$E(15) = 255(15) + 6559$
$E(15) = 3825 + 6559$
$E(15) = 10,384$

The elevation of the tram at the end of the ride (15 minutes) is 10,384 feet.

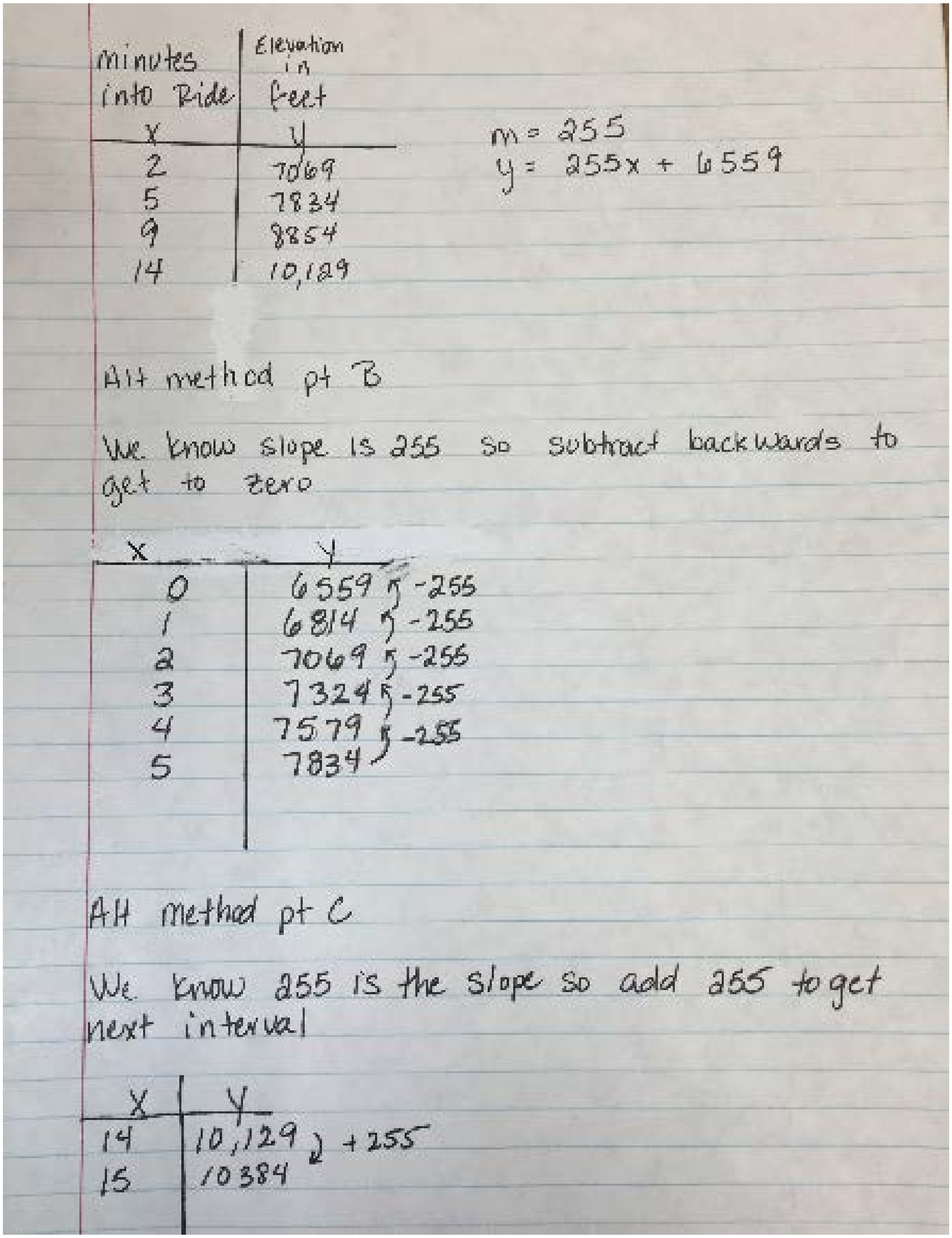
Part b: Why did you use 0 (zero) for the time? What does the point (0, 6559) represent in context of the problem? Where would this point be on the coordinate plane if you were to graph the data? What other methods could you use to solve the problem?

Part c: How did you decide where to substitute 15 into the equation? What does the point (15, 10384) represent in context of the problem? Is this predicted point an interpolation or an extrapolation? How do you know? What other methods could you use to solve the problem?

Alternative Strategy 1



Alternative Strategy 2



minutes into Ride x	Elevation in feet y
2	7069
5	7834
9	8854
14	10,129

m = 255
y = 255x + 6559

Alt method pt B

We know slope is 255 so subtract backwards to get to zero

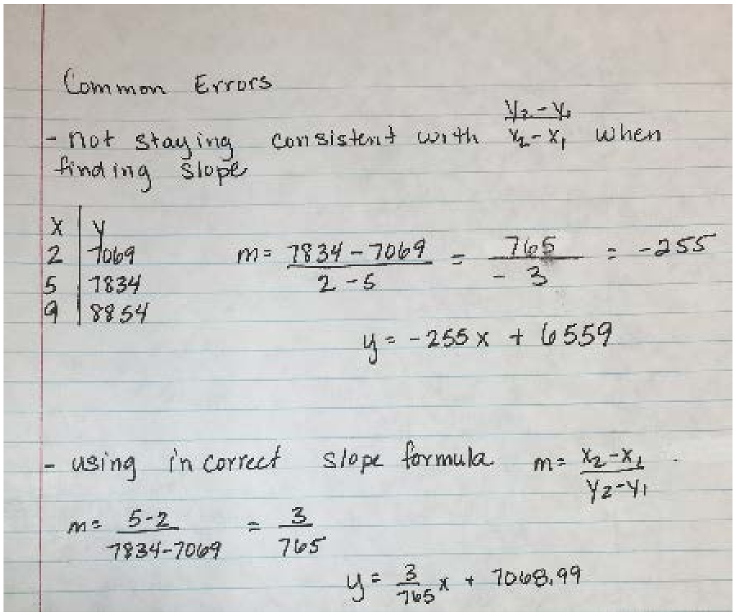
X	Y
0	6559
1	6814 -255
2	7069 -255
3	7324 -255
4	7579 -255
5	7834 -255

Alt method pt C

We know 255 is the slope so add 255 to get next interval

X	Y
14	10,129 +255
15	10384

Common Incorrect Response



What is the slope formula? Are you using the correct formula for slope? Does your answer make sense based on what is happening as the tram moves forward? Does having a negative slope make sense given that the elevation is increasing?

Debrief

Whole/Large Group Discussion

- Debriefing formats may differ (e.g., whole-class discussion, small-group discussion). It will be beneficial for students to view student work as a gallery walk or similar activity.
- Have students/teacher facilitate the sequence of multiple representations in an order that moves from less to more mathematical sophistication.
- Allow students to question each other and explain their choices, using mathematical reasoning. If students struggle, use questioning strategies.
- Encourage students to notice similarities, differences, and generalizations across strategies.
- Provide constructive feedback and ask clarifying questions for deeper understanding of the process.

If you see this common error…, it might mean this…

Student has a negative slope….the student exchanged y_2 and y_1, (or x_2 and x_1) when calculating slope.

Student has a proper fraction for slope…the student used the reciprocal of the slope formula.

The linear equation is incorrect…the student substituted x_1 and y_1 incorrectly when writing point-slope form.

The linear equation is incorrect…the student forgot to distribute the slope to both terms when transforming point-slope form to slope-intercept form.

The linear equation is incorrect…the student has confused the independent and dependent variables.
The student has chosen the wrong equation type…the student did not determine there is a constant rate of change and/or did not recognize a common difference on the table since it does not increase by 1 for input values.
The student has the initial elevation incorrect…the student does not understand that the starting time is 0.

Synthesize and Apply

Monitor student work and facilitate discussions by asking questions. When students have independently arrived at the Enduring Understanding, engage them in solving these extension problems. Assess if you have facilitated the discussion in a way that students have arrived at the Enduring Understanding (do not tell them, they will benefit from discovering it for themselves).

Extension Problem #1

Sam boards a ski lift at *Breckenridge Ski Resort* in Colorado. He rides up the mountain at eight feet per second. Sam starts his journey at an elevation of 8755 ft.

Part a: Write an equation for the function (linear, quadratic, or exponential) that models the relationship between the elevation of the ski lift and the number of seconds into the ride.

Part b: Create a table of values for the elevation at 12, 32, 75 and 136 seconds.

Part c: How many seconds will it take Sam to reach the top of the mountain if the elevation at the top is 10,677ft? How many minutes is this, approximately?

Possible Solution:

Part a: m = 8 ft/second y-int = 8755 ft Using $y= mx + b$ the equation is $y = 8x + 8755$

Part b:

Seconds on Lift (x)	Elevation (y)
12	8851
32	9011
75	9355
136	9843

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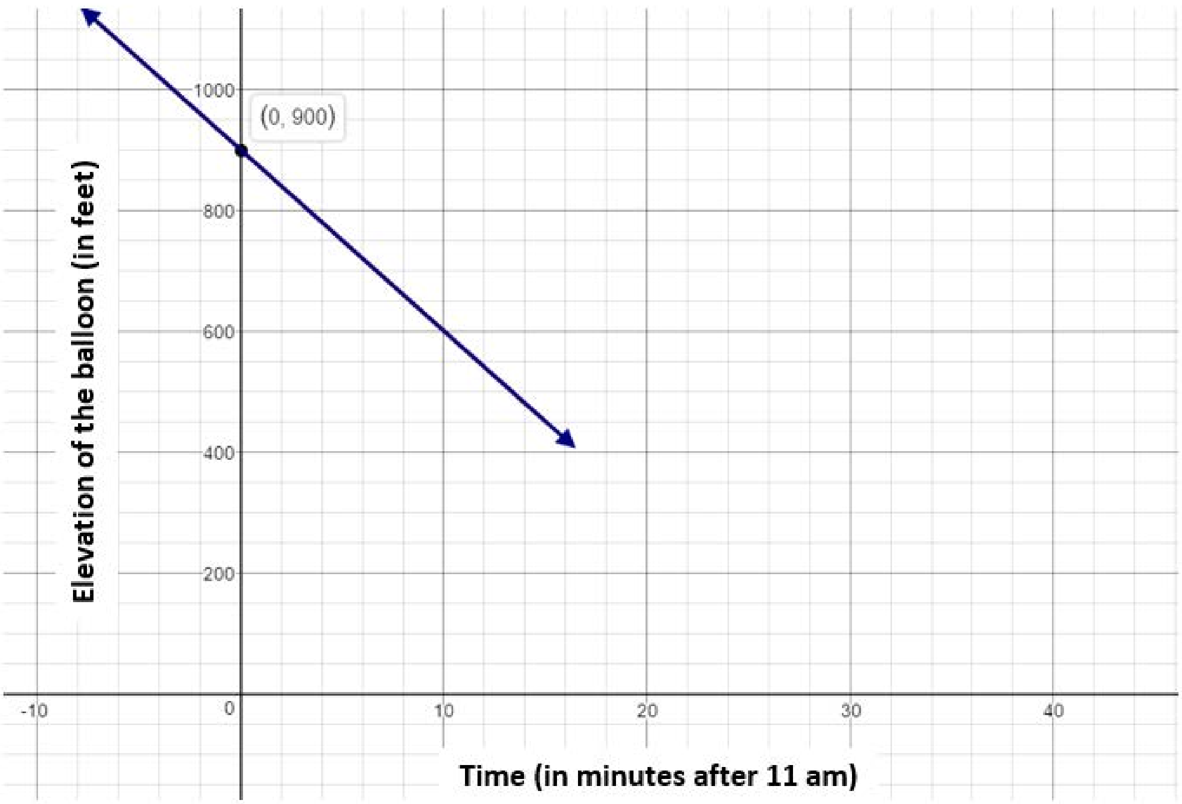
Part c: Ending elevation is 10,677 ft. Using $y = 8x + 8755$, find x when $y = 10{,}677$.
$10{,}677 = 8x + 8755$, $x = 240.25$ seconds, which is approximately 4 minutes.

Extension Problem #2

The *Albuquerque International Balloon Fiesta* is a festival of hot air balloons that is held annually in New Mexico. Last year, one of the colorful balloons was spotted at an altitude of 900 feet at 11 am. The graph below shows the path of the balloon as it descended.



- **a.** Create a table of values for the situation with at least 3 different times and elevations.
- **b.** What is the rate of descent in feet per minute?
- **c.** Write an equation for a function (linear, quadratic, or exponential) that models the relationship between the elevation of the balloon and the time in minutes after 11 am.
- **d.** How high was the balloon 3 minutes before it was sighted?
- **e.** How high was the balloon at 11:24 am?
- **f.** How long will it take the balloon to descend to an altitude of 420 feet?
- **g.** At what time will the balloon land? Possible Solution:

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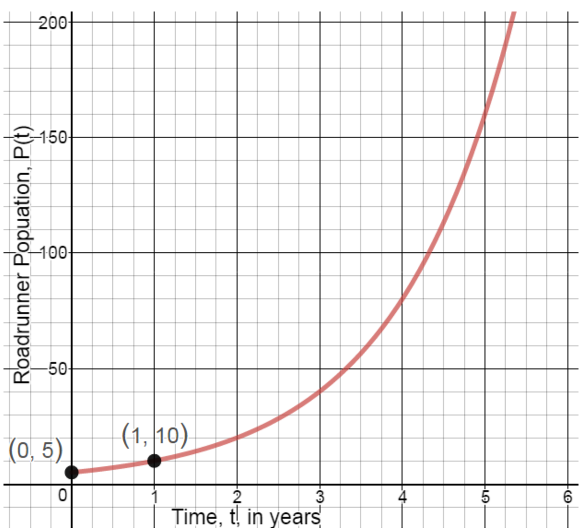
a. Answers may vary. Table may include:

Time (in minutes)	Elevation (in feet)
0	900
5	750
10	600
15	450
20	300
25	150
30	0

b. The rate of descent is 30 feet per minute.
c. Linear, $E(t) = 900 - 30t$
d. 990 feet; $E(-3) = 900 - 30(-3) = 990$
e. 180 feet; $E(24) = 900 - 30(24) = 180$
f. 16 minutes; $420 = 900 - 30t$
g. 11:30 am; $0 = 900 - 30t$, time = 30 minutes

Extension Problem #3



Roadrunners are native to the Southwest U.S. and Mexico. The following graph represents the population of roadrunners in one area. The x-axis is t, the time in years, and the y-axis is $P(t)$, the roadrunner population for any given year. In this case, $t = 0$, indicates the year they started keeping track of the roadrunner population. Scientists want to know how the population is changing every two years so they can make predictions about the future.

Part 1: Analyzing the Graph

Is the graph increasing or decreasing? What does this mean in terms of the roadrunner population? What is the horizontal asymptote of the graph? Identify the domain and range and explain what they represent in terms of the roadrunner population (think of positive, negative, continuous, discrete, etc). What is the y-intercept? What does this mean in terms of the roadrunner population? What is the point on the graph when x = 1? What does this mean in terms of the roadrunner population?

Part 2: Writing the Function

The graph shows an exponential relationship of the form $P(t) = a \cdot b^t$. What is the function that represents the roadrunner population?

Part 3: Using the Function

Use the function to estimate the roadrunner population in years 2, 3, and 4. Do you notice any patterns among years 2, 3, and 4? How is the roadrunner population changing every year? How is the roadrunner population changing every two years?

Part 4: Making Predictions

Predict the number of roadrunners in year 10. Predict the number of roadrunners in year 15. Do you think the roadrunner population could become a problem? In other words, do you think the population will continue to grow in this way forever? Why or why not?

Possible Solution:

Part 1: Analyzing the Graph

The graph is increasing. This means the roadrunner population is growing. The horizontal asymptote is $y = 0$. The domain is $x \geq 0$, which means the time, in years, must be greater than or equal to zero. Time is continuous. The range is $y \geq 5$, which means the population must be greater than or equal to the starting population of 5 roadrunners. The population is discrete since you can only have whole numbers to represent the roadrunner population. The y-intercept is 5 since that is the starting population in year 0. The point on the graph when x = 1 is (1, 10). This means that after 1 year, there are 10 roadrunners.

Part 2: Writing the Function

The function that represents this model is: $P(t) = 5 \cdot 2^t$, where $a = 5$ represents the starting population, and $b = 2$ represents the rate of change from year 0 to year 1.

Part 3: Using the Function

The population from year 2 to year 3 is multiplied by 2; from year 3 to year 4 is also multiplied by 2. This means the roadrunner populations is doubling each year. This means the population is being multiplied by 4 (or quadrupling) every two years.

Years	Population
2	20 roadrunners
3	40 roadrunners
4	80 roadrunners

Part 4: Making Predictions

There will be 5120 roadrunners in year 10. There will be 163,840 roadrunners in year 15. The population will not grow this way forever. Students should provide answers that may include biological and environmental constraints (such as food supply, natural predators, climate/weather changes, illness/death, etc).

References

Common Core State Standards Initiative. (2010). *Common core state standards for mathematics.* Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.

Illustrative Mathematics

Polya, G. (2014). How to solve it: *A new aspect of mathematical method.* Princeton, NJ: Princeton University Press.

Name______________________________

Student Page

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