

# Find a System

## Enduring Understanding

**(Do not tell students; they must discover it for themselves.)**

Students will develop further understanding of the solution to a system of linear equations both by finding the solution and by finding the system given constraints.

### Standards

HSA.REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

HSA.CED.A.2 Create equations in two or more variables to represent relationships between quantities and graph equations on coordinate axes with labels and scales. Limit exponentials to have integer inputs only. \*(Modeling Standard)

## Launch

### Introduce the Task

Construct a system of two linear equations where  $(-2, 3)$  is a solution to the first equation but not to the second equation, and where  $(5, -2)$  is a solution to your system.

Explain how your system satisfies the required conditions.

### Understand the Problem

- Are there any word(s) you don't understand?
- What is the question or task asking you to answer?
- Is there enough information to find a solution?
- Restate the problem in your own words.
- What additional information do you need to find?

## Develop a Plan

- There are many reasonable ways to solve a problem. With practice, students will build the necessary skills to choose an efficient strategy for the given problem.
- Ensure that students have a place to start and that the task/problem has the ability to be scaffolded.
- Caution should be exercised to not force your plan/reasoning on students.

## Investigate

### Productive Struggle

- Let students engage in productive struggle.
- Monitor as students work.
- Offer positive constructive feedback.
- Ask questions such as...
  - Why did you choose that number?
  - What assumptions did you make?
  - Explain what you are doing here.
  - What does that solution mean?

### Questions for Individuals as they Work

**Students are unable to start the problem...** What do we know? What do we need to know? What is an equation for a line? What does it mean to be a solution to a system? What methods have you tried? What other methods could you use? (Tables, Graphs, Equations)

**The student is having difficulty finding slope ...** What two points should you use? What is the definition of slope?

**The student is struggling to initially create an equation ...** What do you need to know to write the equation of a line? What are the different forms of linear equations? How do you find the y-intercept?

**The student is struggling with a fraction ...** Is there a way to write an equivalent equation without fractions? Is it necessary to rearrange the equation? Why?

**The Student is finished ...** Did you verify that your system meets the given requirements?

# Sample Solutions

## Possible Correct Response

$x$	-2	0	5
$y_1$	3	-2	
$y_2$	Any # but 3	-2	

$x$	-2	5
$y_1$	3	-2
		-5

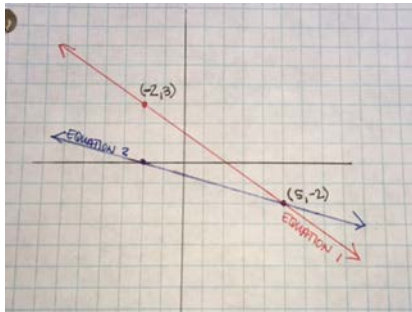
$\Delta y = -5$   
 $\Delta x = 7$

Eqn 1:  $y+2 = -\frac{5}{7}(x-5)$

Eqn 2:  $y+2 = -\frac{3}{7}(x-5)$

Ex.  $x$  | -2 5  
 $y_2$  | 0 -2  
          -2

any #  
but 3



Why did you choose this form of a linear equation? What does that solution mean? Did you answer all parts of the question? Is there more than one method to find your equations? What are they? Will everyone have the same equations in the system? Why or why not?

$m = \frac{-2-3}{5-(-2)} = -\frac{5}{7}$

$y+2 = -\frac{5}{7}(x-5)$

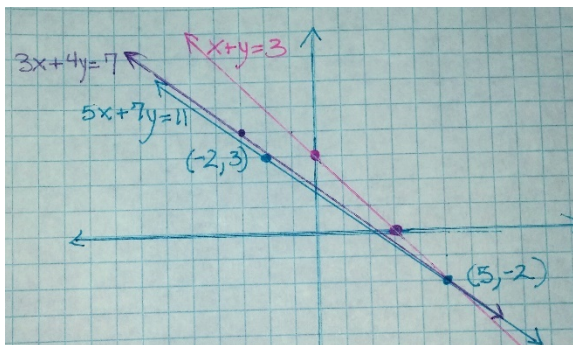
$7(y+2) = -5(x-5)$

$7y+14 = -5x+25$

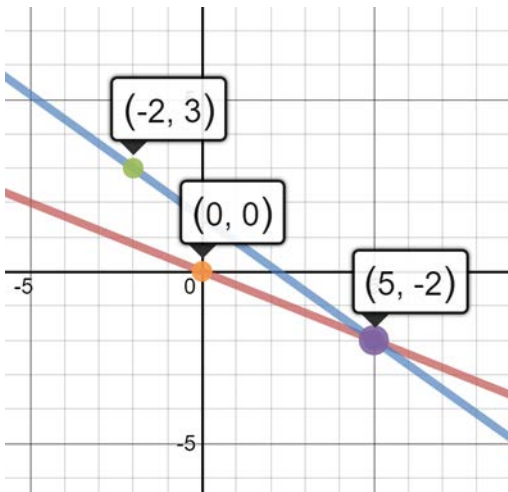
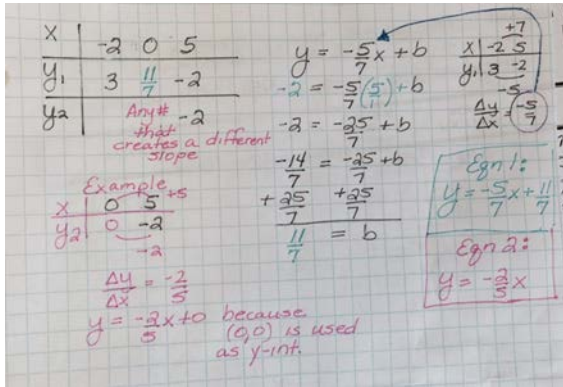
EQ1:  $5x+7y = 11$

EQ2:  $x+y = 3$   
 $5+(-2) = 3$   
 $x+y = 3$

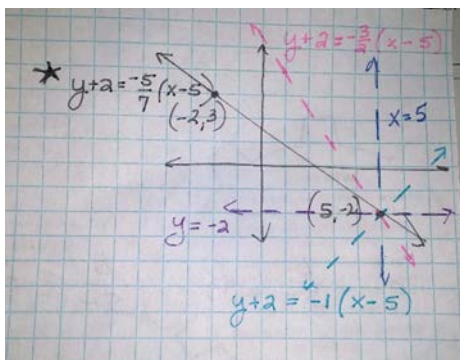
or  $A(5) + B(-2) = C$   
 $3(5) + 4(-2) = C$   
 $15 - 8 = C$   
 $7 = C$   
 $3x + 4y = 7$



Why did you choose this form of a linear equation? What does that solution mean? Did you answer all parts of the question? Is there more than one method to find your equations? What are they? Will everyone have the same equations in the system? Why or why not?

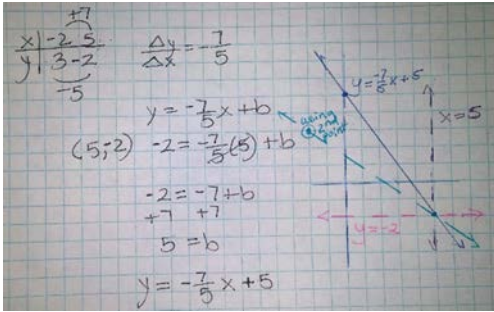


Why did you choose this form of a linear equation? What does that solution mean? Did you answer all parts of the question? Is there more than one method to find your equations? What are they? Will everyone have the same equations in the system? Why or why not?



The starred equation is equation 1. Equation 2, answers can vary. What makes equation 2 work? How does the graph satisfy the required conditions? Is there more than one method to find your equations? What are they? Will everyone have the same equations in the system? Why or why not?

## Common Incorrect Response



Student inverted slope. The second line contains  $(-2, 3)$  when that point should be on the first line.

## Debrief

### Whole/Large Group Discussion

- Debriefing formats may differ (e.g., whole-class discussion, small-group discussion). It will be beneficial for students to view student work as a gallery walk or similar activity.
- Have students/teacher facilitate the sequence of multiple representations in an order that moves from less to more mathematical sophistication.
- Allow students to question each other and explain their choices, using mathematical reasoning. If students struggle, use questioning strategies.
- Encourage students to notice similarities, differences, and generalizations across strategies.
- Provide constructive feedback and ask clarifying questions for deeper understanding of the process.

### If you see this common error..., it might mean this...

If a student answer is:

$$m = -\frac{5}{7} \quad y - 2 = -\frac{5}{7}(x + 5)$$

Review point-slope formula:

$$y - y_1 = m(x - x_1)$$

If a student answer is:

$$m = -\frac{5}{7} \quad y - 5 = -\frac{5}{7}(x + 2)$$

Review ordered pairs.  $x$  and  $y$  are placed in the wrong spot.

## Synthesize and Apply

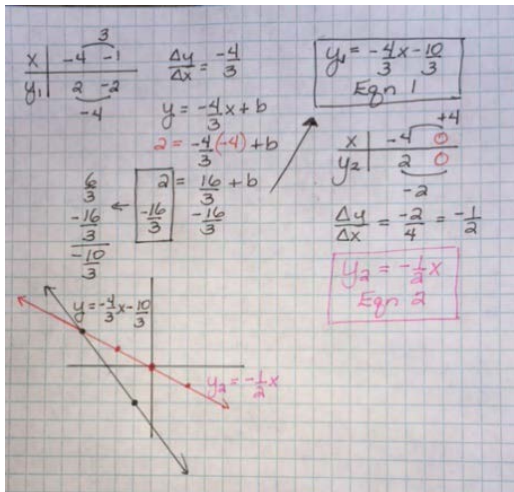
Monitor student work and facilitate discussions by asking questions. When students have independently arrived at the Enduring Understanding, engage them in solving these extension problems. Assess if you have facilitated the discussion in a way that students have arrived at the Enduring Understanding (do not tell them, they will benefit from discovering it for themselves).

### Extension Problem #1

Without graphing, construct a system of two linear equations where  $(-1, -2)$  is a solution to the first equation but not to the second equation, and where  $(-4, 2)$  is a solution to your system.

After you have created your system of equations, graph your system. Explain how your graph shows that your system satisfies the required condition.

#### Possible Solutions:



### Extension Problem #2

Sue is planning to book a vacation rental. She noticed that vacation rentals charge a booking fee plus a nightly charge. She found the following:

Rental A			
Number of nights	3	7	14
Total charge	698	1522	2964
Rental B			
Number of nights	3	7	14
Total charge	738	1482	2784

Use the tables to determine between what days the total charge will be the same. Justify your reasoning.

For what number of nights would it be the best deal to stay at Rental A? Or Rental B? Explain.

## Possible Solution:

A	<table style="border-collapse: collapse; width: 100%;"> <tr> <td style="padding: 2px 10px;">3</td> <td style="padding: 2px 10px;">7</td> <td style="padding: 2px 10px;">14</td> </tr> <tr> <td style="padding: 2px 10px;">698</td> <td style="padding: 2px 10px;">1522</td> <td style="padding: 2px 10px;">2964</td> </tr> </table>	3	7	14	698	1522	2964	$m = \frac{1522 - 698}{7 - 3} = \frac{824}{4} = 206$
3	7	14						
698	1522	2964						
B	<table style="border-collapse: collapse; width: 100%;"> <tr> <td style="padding: 2px 10px;">3</td> <td style="padding: 2px 10px;">7</td> <td style="padding: 2px 10px;">14</td> </tr> <tr> <td style="padding: 2px 10px;">738</td> <td style="padding: 2px 10px;">1482</td> <td style="padding: 2px 10px;">2784</td> </tr> </table>	3	7	14	738	1482	2784	$m = \frac{1482 - 738}{7 - 3} = \frac{744}{4} = 186$
3	7	14						
738	1482	2784						

1) Between days 3 and 7, there will be a time when the total price of rental A will be equal to rental B.

reasoning: on day 3 rental A is less than rental B but on day 7 rental A is more than rental B.

2) A:  $y - 698 = 206(x - 3)$   
 $y = 206x - 618 + 698$   
 $y = 206x + 80$

B:  $y - 738 = 186(x - 3)$   
 $y = 186x - 558 + 738$   
 $y = 186x + 180$

$$206x + 80 = 186x + 180$$

$$20x = 100$$

$$x = 5$$

on day 5 the prices are the same that price is \$ 1110

$$206(5) + 80 = 186(5) + 180$$

$$1110 = 1110$$

Best deals
less than 5 nights at Rental A
more than 5 nights at Rental B

## Extension Problem #3

Haden has recently been offered a raise at his part-time job as a salesperson at a local clothing store. He has a choice of three different pay scales. The first option is to receive a base salary of \$350 a week plus 10% of the price of the merchandise he sells. The second option is a base salary of \$230 a week plus 25% of the price of the merchandise he sells. The third option is a base salary of \$100 a week and 30% of the price of the merchandise he sells. For what range of sales will option 2 be the best option for one work week?

## Possible Solutions:

\$800 < sales < \$2600

## References

Common Core State Standards Initiative. (2010). *Common core state standards for mathematics*. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.

### [Illustrative Mathematics](#)

Polya, G. (2014). *How to solve it: A new aspect of mathematical method*. Princeton, NJ: Princeton University Press.





Name \_\_\_\_\_

## Student Page

Construct a system of two linear equations where  $(-2,3)$  is a solution to the first equation but not to the second equation, and where  $(5,-2)$  is a solution to your system.

Explain how your system satisfies the required conditions.

