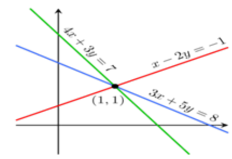
EOC Math I Remediation Lessons

EOC I with a Focus on Algebra



Speed Trap

Enduring Understanding

(Do not tell students; they must discover it for themselves.)

Students will create and interpret graphical displays. Students will interpret and compare data distributions.

Standards

HSS.ID.A.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).
HSS.ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
HSS.ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Launch

Introduce the Task

A statistically-minded state trooper wondered if the speed distributions are similar for cars traveling northbound and for cars traveling southbound on an isolated stretch of interstate highway. He uses a radar gun to measure the speed of all northbound cars and all southbound cars passing a particular location during a fifteen minute period. Here are his results:

Northbound Cars					Southbound Cars				
60	62	63	63	63	55	56	57	57	58
63	64	64	64	65	60	61	61	62	63
65	65	65	66	66	64	65	65	67	67
67	68	70	83		68	68	68	68	71

Draw box plots of these two data sets and then use the plots and appropriate numerical summaries of the data to write a few sentences comparing the speeds of northbound cars and southbound cars at this location during the fifteen minute time period.

Describe a situation that could lead to the differences in the observed data.

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Understand the Problem

- Are there any word(s) you don't understand?
- What is the question or task asking you to answer?
- Is there enough information to find a solution?
- Restate the problem in your own words.
- What additional information do you need to find?

Develop a Plan

- There are many reasonable ways to solve a problem. With practice, students will build the necessary skills to choose an efficient strategy for the given problem.
- Ensure that students have a place to start and that the task/problem has the ability to be scaffolded.
- Caution should be exercised to not force your plan/reasoning on students.

Investigate

Productive Struggle

- Let students engage in productive struggle.
- Monitor as students work.
- Offer positive constructive feedback.
- Ask questions such as…
  - Why did you choose that number?
  - What assumptions did you make?
  - Explain what you are doing here.
  - What does that solution mean?

Questions for Individuals as they Work

Students are unable to start the problem.… What do you know about box plots? Are you familiar with the five number summary?

The student is unable to find the median of an even numbered list of entries. … Do you know how to find the average of two values? How do you find the value that is in-between two numbers?

The student is calculating the mean. … What are the other measures of central tendency?

The student is scaling the number line incorrectly.… How does the range of the data set help us determine scaled units on a number line?

The student has combined the data into one set…How was the question worded? Are you able to compare with only one data set?

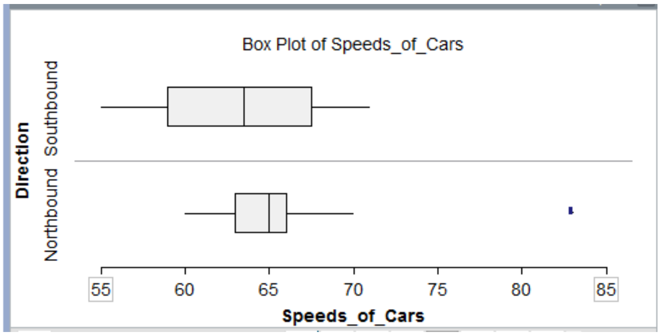
The student is unable find a quartile value.… How are you organizing your data set? How did you find the median value? Did you include the median when finding the quartile values?

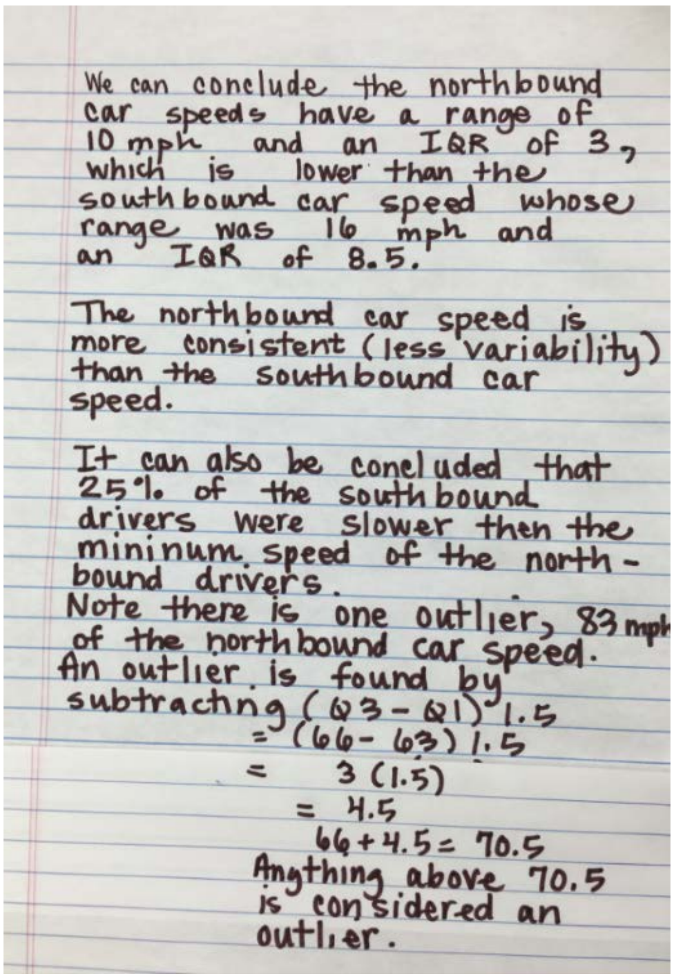
The student is confusing the interquartile range (IQR) with the range… What is the difference between the two measures of spread?

The student is using range to compare variability. … What does the range tell you about the data sets? What are other measures of variability?

Sample Solutions

Possible Correct Response





We can conclude the northbound car speeds have a range of 10 mph and an IQR of 3, which is lower than the southbound car speed whose range was 16 mph and an IQR of 8.5.

The northbound car speed is more consistent (less variability) than the southbound car speed.

It can also be concluded that 25% of the southbound drivers were slower then the mininum speed of the north-bound drivers.

Note there is one outlier, 83 mph of the northbound car speed.

An outlier is found by subtracting (Q3 − Q1) 1.5

= (66 − 63) 1.5

= 3 (1.5)

= 4.5

66 + 4.5 = 70.5

Anything above 70.5 is considered an outlier.

How did you get this answer? Why is 83 disconnected from the rest of your plot? Why did you put the box plots over the same number line? How would your graph have been affected had you done this without graph paper? Which value is best for discussing variability? How did you know what to describe in regards to the shape of the box plots? Did you address the shape, outliers, center, and spread (SOCS)? What were the different descriptive words that you used to compare the box plots?

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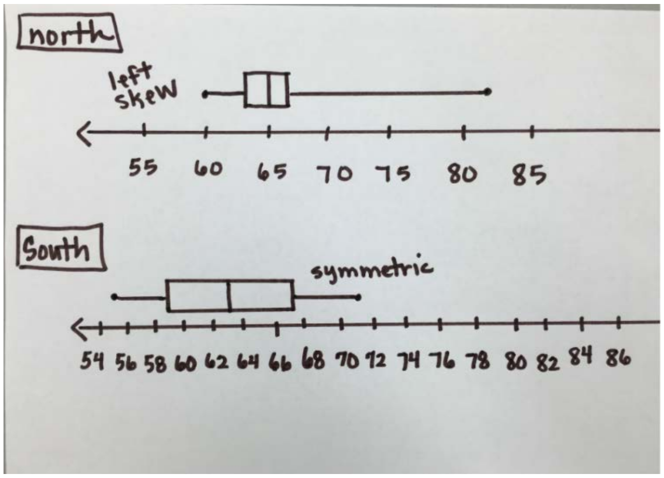
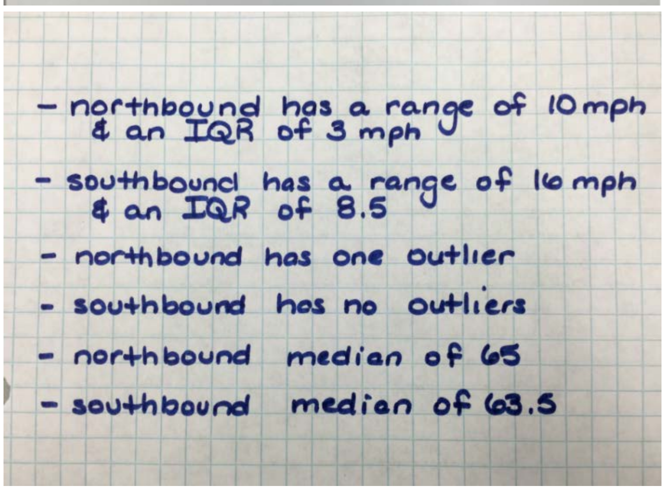
What does the range help you determine in regards to the data sets? Situations must be mathematically justified.

Common Incorrect Response





How many data items are between 66 and 83? What kind of effect does including 83 have on the distribution? How do we determine the direction of skewness of a distribution? (The northbound car speeds is skewed right). Would it be more helpful to compare the box plots when they are on the same number line? Are you comparing speeds or are you just listing attributes? What are the attributes that we should discuss when describing a distribution? What were the different descriptive words that you used to compare the data?

Debrief

Whole/Large Group Discussion

- Debriefing formats may differ (e.g., whole-class discussion, small-group discussion). It will be beneficial for students to view student work as a gallery walk or similar activity.
- Have students/teacher facilitate the sequence of multiple representations in an order that moves from less to more mathematical sophistication.
- Allow students to question each other and explain their choices, using mathematical reasoning. If students struggle, use questioning strategies.
- Encourage students to notice similarities, differences, and generalizations across strategies.
- Provide constructive feedback and ask clarifying questions for deeper understanding of the process.

If you see this common error…, it might mean this…

Students included the outlier of 83 in the box plot as a maximum value… The student may not know the formula to determine whether a value is an outlier or not. Formulas are:
Small values $x<Q_1-1.5*IQR$
Large values $x>Q_3+1.5*IQR$

Students are only listing data summaries rather than comparing them... The student is not writing the conclusions of the southbound car speeds in comparison to the northbound car speeds.

Students are not defining the variables correctly in the conclusion. (Ex. Using northbound rather than northbound car speeds)… When drawing conclusions, the student might not know that a variable is represented by its quantitative units rather than categorical units.

Students are graphing the box plots under two separate number lines that might be scaled differently… Student may not know how to properly compare graphical displays.

Students are starting their number lines at zero… Student may not understand proper scaling of data.

Students describe the skewedness of the distribution incorrectly… Student may not know how to label the direction of skewed data in the direction it is being pulled.

Synthesize and Apply

Monitor student work and facilitate discussions by asking questions. When students have independently arrived at the Enduring Understanding, engage them in solving these extension problems. Assess if you have facilitated the discussion in a way that students have arrived at the Enduring Understanding (do not tell them, they will benefit from discovering it for themselves).

Extension Problem #1

A math teacher collected data from their first and last period classes. The teacher wants to compare the results of a recent test. The results are as follows:

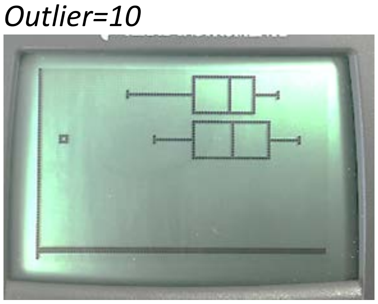
First class test scores: 34, 42, 43, 49, 60, 60, 60, 68, 73, 73, 77, 80, 82, 82, 83, 88, 90, 91, 92

Last class test scores: 10, 45, 52, 60, 60, 60, 67, 67, 68, 73, 75, 75, 80, 80, 86, 91, 91, 94, 97, 100

Draw box plots of these two data sets, then use the plots and appropriate numerical summaries of the data to write a few sentences comparing the test results of the two classes.

Possible Solution:

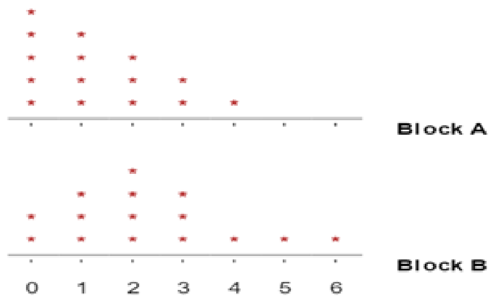
first class: min=34
last class: min=10
Q_1=60 Q_1=60
med=73; med=74
Q_3=83; Q_3=88.5
max=92; max=100
Outlier=10



We can conclude that the students in the last class scored slightly better than those in the first class. The first class test scores are skewed left while the last class test scores are more symmetric. Although quartile 1 is the same value in both sets of test scores, the IQR is larger for the test scores in the last class.The median test scores were very close in both classes, but the test scores of the last class had a slightly higher median. The last class has an outlier of 10 which would affect the mean score, but not the median. Therefore the median is a better measure of center.

Extension Problem #2

Look at the following dot plots. The dot plots show pet ownership in two city blocks in Carson City.



Part I: Use the plots to find the appropriate numerical summaries.
Part II: Write a few sentences comparing the pet ownership on Block A and Block B. Describe a situation that could justify the obsered data set.
Part III: Knowing that data can be represented in a variety of ways, discuss the most appropriate measure of center in regards to these data sets. (Ex. mean, median, and mode). Why is your answer the most appropriate?
Part IV: Which data set has the larger standard deviation? Explain your reasoning.

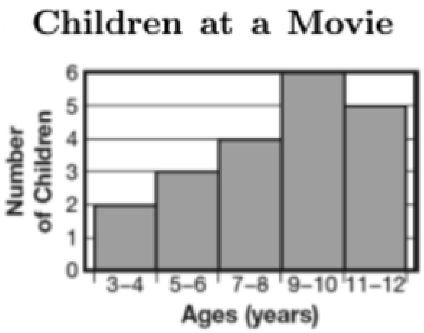
Possible Solution:

Part I: Block A: min=0 Block B: min=0
Q_1=0; Q_1=1
med=1; med=2
Q_3=2; Q_3=3
max=4; max=6

Part II: We can conclude that more pet lovers live on Block B than on Block A since the median of pets owned on Block B is twice as much as the median of pets owned on Block A. The spread is larger on the number of pets owned by individuals on Block B. The distribution of Block A of pets owned is more skewed right than Block B pets owned representing more zeros of pet ownership. Situations must be mathematically justified.
Part III: The median value would be the most appropriate to use when comparing skewed data sets.
Part IV: Block B would have the larger standard deviation since the number of pets is more spread out.

Extension Problem #3

The following histogram represents 20 summer camp children who attended a movie



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Part I: Where is the mean age in relation to the median age of the children at the movie?
Part II: In regards to the variability, would it be the same if the data was presented in a different graphical display (ex. dot plot, box plots, etc.)?
Part III: If the 22 year old camp counselor was also reported in the survey, how would this affect the mean value of ages? How would this affect the median value of ages?
Part IV: How would the standard deviation of ages be affected?

Possible Solution:

Part I: The mean value of ages is smaller than the median since the histogram is skewed left.
Part II: The variability should remain the same despite the graphical display.
Part III: The mean should increase, but the median would be relatively the same.
Part IV: The standard deviation would increase since the mean and standard deviation are greatly affected by outliers.

References

Common Core State Standards Initiative. (2010). *Common core state standards for mathematics.* Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.

Illustrative Mathematics

Polya, G. (2014). How to solve it: *A new aspect of mathematical method.* Princeton, NJ: Princeton University Press.

Name______________________________

Student Page

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