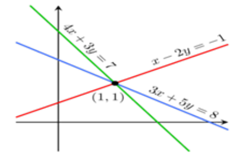
EOC Math I Remediation Lessons

EOC I with a Focus on Algebra



Seeing Dots

Enduring Understanding

(Do not tell students; they must discover it for themselves.)

Students will write, evaluate, and analyze equivalent expressions from models and real-world situations.

Standards

HSA.SSE.A.2 Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$, *thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.
HSA.SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*
HSF.IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

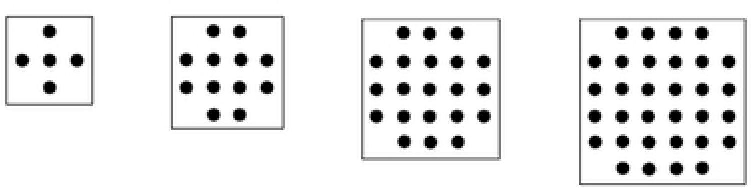
Launch

Introduce the Task

Consider the algebraic expressions below:

$$(n+2)^2 - 4 \text{ and } n^2 + 4n.$$

a) Use the figures below to illustrate why the expressions are equivalent:



b) Verify your results algebraically.

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Understand the Problem

- Are there any word(s) you don't understand?
- What is the question or task asking you to answer?
- Is there enough information to find a solution?
- Restate the problem in your own words.
- What additional information do you need to find?

Develop a Plan

- There are many reasonable ways to solve a problem. With practice, students will build the necessary skills to choose an efficient strategy for the given problem.
- Ensure that students have a place to start and that the task/problem has the ability to be scaffolded.
- Caution should be exercised to not force your plan/reasoning on students.

Investigate

Productive Struggle

- Let students engage in productive struggle.
- Monitor as students work.
- Offer positive constructive feedback.
- Ask questions such as…
  - Why did you choose that number?
  - What assumptions did you make?
  - Explain what you are doing here.
  - What does that solution mean?

Questions for Individuals as they Work

Students are unable to start the problem…What information from the images could be useful? What patterns do you notice? Where do your input values go? What do you notice about the figures and the expressions? Can you illustrate the next figure? Would a table be helpful?

The student is trying to determine if it is an arithmetic or geometric sequence … If it is not an arithmetic or geometric sequence, what other functions might apply?

The student draws the next illustration, but cannot go any further… Does knowing the next picture help you compare the expressions? What other kinds of functions do you know about?

The student fills in the dots to complete the squares, but cannot go any further… What can completing the square tell us about the expressions being equivalent or not?

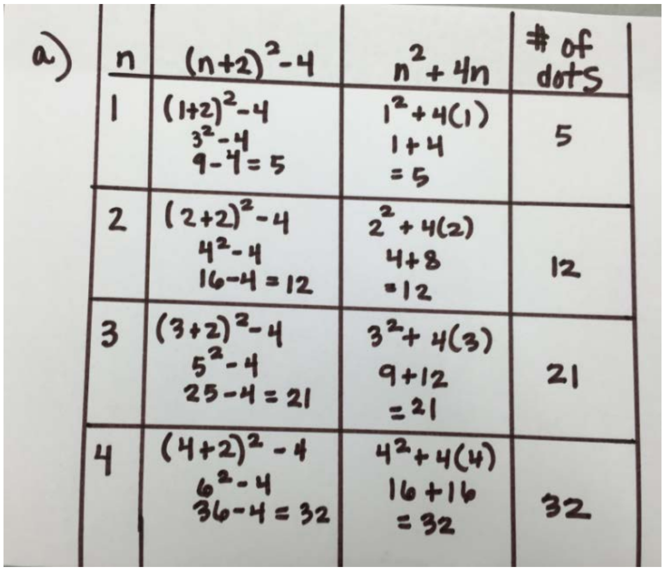
The student has combined the data into one set…How was the question worded? Are you able to compare with only one data set?

The student substitutes into "n" the number of dots in the illustration… What does "*n*" represent?

The student does not set expressions equal to each other… How do you show that two expressions are equivalent?

Sample Solutions

Possible Correct Response

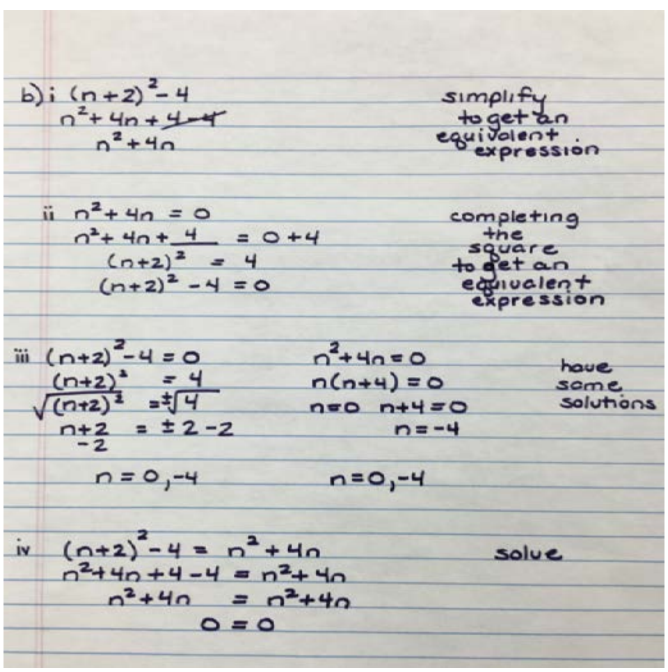


a)

n	$(n+2)^2-4$	n^2+4n	# of dots
1	$(1+2)^2-4$ 3^2-4 $9-4=5$	$1^2+4(1)$ $1+4$ $=5$	5
2	$(2+2)^2-4$ 4^2-4 $16-4=12$	$2^2+4(2)$ $4+8$ $=12$	12
3	$(3+2)^2-4$ 5^2-4 $25-4=21$	$3^2+4(3)$ $9+12$ $=21$	21
4	$(4+2)^2-4$ 6^2-4 $36-4=32$	$4^2+4(4)$ $16+16$ $=32$	32

Part A:

Where did you get the value of *n*? Why did you put this information in a table? How do the outputs from the table show that the expressions are equivalent? How does this prove that the expressions are equivalent?



b) i $(n+2)^2-4$

$n^2+4n+\cancel{4-4}$

n^2+4n

simplify to get an equivalent expression

ii $n^2+4n=0$

$n^2+4n+\underline{4} = 0+4$

$(n+2)^2 = 4$

$(n+2)^2-4=0$

completing the square to get an equivalent expression

iii $(n+2)^2-4=0$

$(n+2)^2 = 4$

$\sqrt{(n+2)^2} = \pm\sqrt{4}$

$\underset{-2}{n+2} = \pm 2 - 2$

$n=0,-4$

$n^2+4n=0$

$n(n+4)=0$

$n=0 \quad n+4=0$

$n=-4$

$n=0,-4$

have same solutions

iv $(n+2)^2-4 = n^2+4n$

$n^2+4n+4-4 = n^2+4n$

$n^2+4n = n^2+4n$

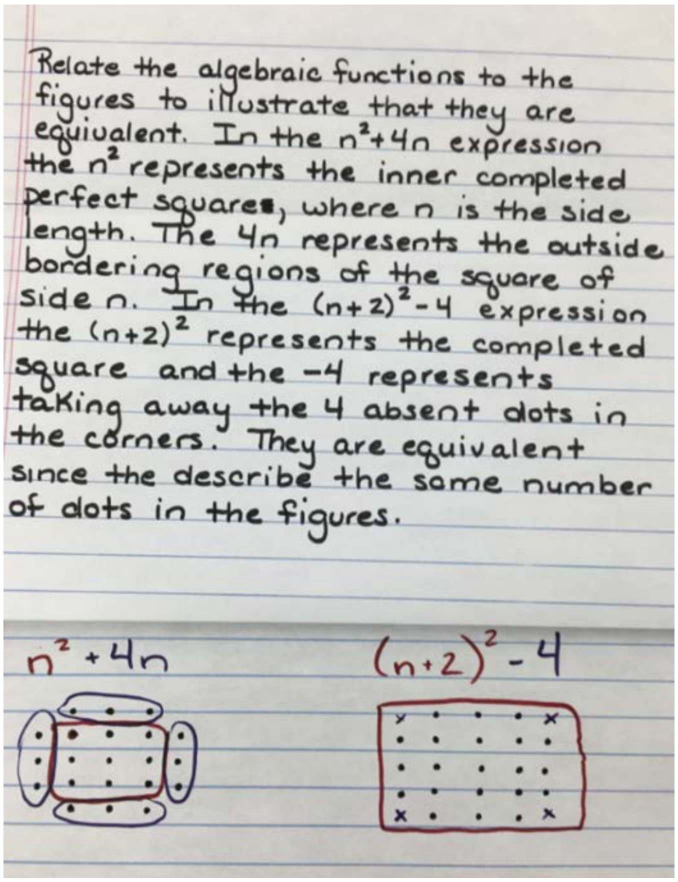
$0=0$

solve

Part B:

What does it mean to simplify? How did you square the binomial? Why did you set it equal to zero? Why did you add 4 to both sides? Why did you set it equal to zero? How did you know to take the square root of both sides? How did you know to factor out the GCF? What does it mean when two equations have the same solutions? Why did you set the expressions equal to each other? What does “0 = 0” mean?

Common Incorrect Response


Relate the algebraic functions to the figures to illustrate that they are equivalent. In the n^2+4n expression the n^2 represents the inner completed perfect squares, where n is the side length. The 4n represents the outside bordering regions of the square of side n. In the $(n+2)^2-4$ expression the $(n+2)^2$ represents the completed square and the −4 represents taking away the 4 absent dots in the corners. They are equivalent since the describe the same number of dots in the figures.

n^2+4n $(n+2)^2-4$

Why did you approach the problem this way? Why did you break the expressions apart? What do the different terms represent

$(n, n^2, 4n,\ (n+2)^2\ -4)$?

How do you know that these expressions are equivalent? Will this work for every case? (Only for the 3rd figure). See if you can create a similar pattern that can be used to find ALL responses.

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Debrief

Whole/Large Group Discussion

- Debriefing formats may differ (e.g., whole-class discussion, small-group discussion). It will be beneficial for students to view student work as a gallery walk or similar activity.
- Have students/teacher facilitate the sequence of multiple representations in an order that moves from less to more mathematical sophistication.
- Allow students to question each other and explain their choices, using mathematical reasoning. If students struggle, use questioning strategies.
- Encourage students to notice similarities, differences, and generalizations across strategies.
- Provide constructive feedback and ask clarifying questions for deeper understanding of the process.

If you see this common error…, it might mean this…

Students substitute in the number of dots in the figure for *n* rather than the position…The student may be confusing the input values with the output values.

Students distribute the exponent incorrectly. Ex: $(n+2)^2 - 4 = n^2 + 4 - 4$… The student may not know how to raise binomials to a power.

When completing the square, students do not set the expression equal to zero… The student does not understand the process of solving a quadratic by completing the square.

Students may have forgotten that $\sqrt{4} = \pm 2$ not just positive 2. $n^2 = 4$ n=±2… The student may not understand that every positive real number has a positive and negative square root.

Students mistake the "0 = 0" for a no solution statement... The student may not understand the meaning of a true statement in equivalent expressions.

Synthesize and Apply

Monitor student work and facilitate discussions by asking questions. When students have independently arrived at the Enduring Understanding, engage them in solving these extension problems. Assess if you have facilitated the discussion in a way that students have arrived at the Enduring Understanding (do not tell them, they will benefit from discovering it for themselves).

Extension Problem #1

Consider the algebraic expressions below:

$n^2 + n + n + 2$ *and* $(n + 2)^2 - 2n - 2$

Part I: Use the figures below to illustrate why the expressions are equivalent:

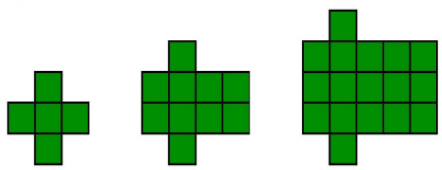


Figure 1 Figure 2 Figure 3

Part II: Find two different ways to algebraically verify the same result.

Possible Solutions:

Part I:

n	$n^2+n+n+2$	$(n+2)^2-2n-2$	# of squares
1	$1^2+1+1+2$ $3+2$ $=5$	$(1+2)^2-2(1)-2$ 3^2-2-2 $9-4$ $=5$	5
2	$2^2+2+2+2$ $4+6$ $=10$	$(2+2)^2-2(2)-2$ 4^2-4-2 $16-6$ $=10$	10
3	$3^2+3+3+2$ $9+8$ $=17$	$(3+2)^2-2(3)-2$ 5^2-6-2 $25-8$ $=17$	17

Part II:

Part II:

i. $(n+2)^2-2n-2$ simplify to get an equivalent expression

$(n+2)(n+2)-2n-2$

$n^2+2n+2n+4-2n-2$

n^2+2n+2

ii. $(n+2)^2-2n-2 = n^2+2n+2$ solve

$n^2+4n+4-2n-2 = n^2+2n+2$

$n^2+2n+2 = n^2+2n+2$

$0=0$

Extension Problem #2

Rich dove off a cliff into Lake Mead while cliff diving with friends. His height as a function of time is modeled by the function $h(x) = -x^2 + 2x + 8$, where *x* is the time in seconds and *h* is the height in meters.

Part I: Write an equivalent expression (in vertex form). How long did it take Rich to reach his maximum height? What was his maximum height?

Part II: Write an equivalent expression (in factored form). How long did it take Rich to hit the water?

Possible Solutions:

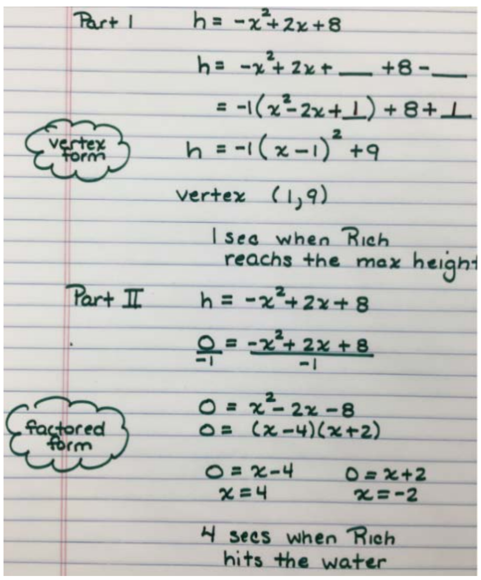
Part I: $h = -(x-1)^2 + 9$

It took 1 second to reach the maximum height.

Maximum height is 9 meters.

Part II: $(x-4)(x+2) = 0$

It took 4 seconds for Rich to hit the water.



Part I

$h = -x^2 + 2x + 8$

$h = -x^2 + 2x + \_\_\_ + 8 - \_\_\_$

$= -1(x^2 - 2x + \underline{1}) + 8 + \underline{1}$

vertex form

$h = -1(x-1)^2 + 9$

vertex $(1, 9)$

1 sec when Rich reaches the max height

Part II

$h = -x^2 + 2x + 8$

$\frac{0}{-1} = \frac{-x^2 + 2x + 8}{-1}$

factored form

$0 = x^2 - 2x - 8$

$0 = (x-4)(x+2)$

$0 = x - 4 \qquad 0 = x + 2$

$x = 4 \qquad x = -2$

4 secs when Rich hits the water

Extension Problem #3

You decide to use chalkboard paint to create a chalkboard on your door. You want the chalkboard to cover 28 ft² and to have a uniform border as shown. Find the width of the border to the nearest inch.

Possible Solution:

x = width in feet

$\text{Area} = L \cdot W$

$L = 8 - 2x$

$W = 4 - 2x$

$A = 28$ ft.

$28 = (8-2x)(4-2x)$

$28 = 32 - \underline{16x} - \underline{8x} + 4x^2$

$$\begin{array}{rl} 28 = & \cancel{32} - 24x + 4x^2 \\ -32 & -\cancel{32} \\ \hline -4 = & -24x + 4x^2 \end{array}$$

$\dfrac{-4 = -24x + 4x^2}{4}$

$-1 = -6x + x^2$

$x^2 - 6x \underline{+9} = -1 + 9$

$\sqrt{(x-3)^2} = \sqrt{8}$

$$\begin{array}{rl} x - 3 = & \pm 2.8 \\ +3 \quad +3 & \\ \hline x = 3 \pm 2.8 & \end{array}$$

$x = 3 + 2.8 = 5.8$ ft

$x = 3 - 2.8 = .2$ ft

It is not possible to use 5.8 ft since width of door is only 4 ft. Use .2 ft and convert to inches.

$.2 \text{ ft} \times 12 \text{ in} = 2.4 \text{ in}.$

The width of the border is 2.4 inches.

References

Common Core State Standards Initiative. (2010). *Common core state standards for mathematics.* Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.

Illustrative Mathematics

Polya, G. (2014). How to solve it: *A new aspect of mathematical method.* Princeton, NJ: Princeton University Press.

Name______________________________

Student Page

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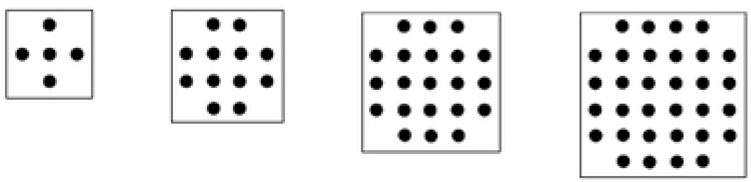
a) Use the figures below to illustrate why the expressions are equivalent:



b) Find some ways to algebraically verify the same result.