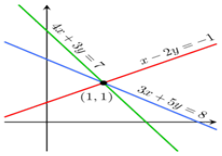


Mathemafish Population

Enduring Understanding

(Do not tell students; they must discover it for themselves.)

Students will find average rates of change over intervals while interpreting algebraic, tabular and graphical forms of functions.

Standards

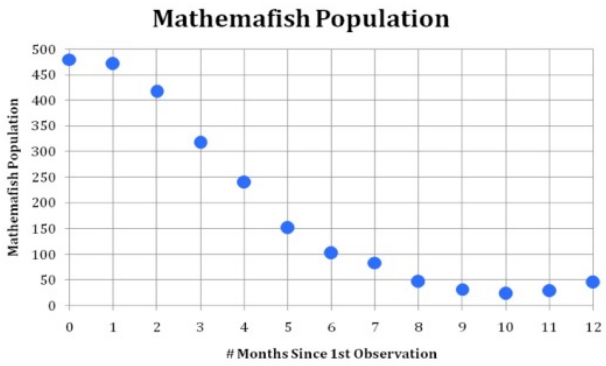
HSF.IF.B.6 Calculate and interpret the average rate of change of a linear, exponential, or quadratic function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph of a function over a specified interval.
HSF.LE.A.1.b Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

Launch

Introduce the Task

You are a marine biologist working for the Environmental Protection Agency (EPA). You are concerned that the rare coral mathemafish population is being threatened by an invasive species known as the fluted dropout shark. The fluted dropout shark is known for decimating whole schools of fish. Using a catch-tag-release method, you collected the following population data over the last year.

# months since 1st measurement	0	1	2	3	4	5	6	7	8	9	10	11	12
Mathemafish population	480	472	417	318	240	152	103	84	47	32	24	29	46



Through intervention, the EPA was able to reduce the dropout population and slow the decimation of the mathemafish population. Your boss asks you to summarize the effects of the EPA's intervention plan in order to validate funding for your project.

What to include in your summary report:

- Calculate the average rate of change of the mathemafish population over specific intervals. Indicate how and why you chose the intervals you chose.
- When was the population decreasing the fastest?
- During what month did you notice the largest effects of the EPA intervention?
- Explain the overall effects of the intervention.
- Remember to justify all your conclusions using supporting evidence.

Understand the Problem

- Are there any word(s) you don't understand?
- What is the question or task asking you to answer?
- Is there enough information to find a solution?
- Restate the problem in your own words.
- What additional information do you need to find?

Develop a Plan

- There are many reasonable ways to solve a problem. With practice, students will build the necessary skills to choose an efficient strategy for the given problem.
- Ensure that students have a place to start and that the task/problem has the ability to be scaffolded.
- Caution should be exercised to not force your plan/reasoning on students.

Investigate

Productive Struggle

- Let students engage in productive struggle.
- Monitor as students work.
- Offer positive constructive feedback.
- Ask questions such as…
    - Why did you choose that number?
    - What assumptions did you make?
    - Explain what you are doing here.
    - What does that solution mean?

Questions for Individuals as they Work

Students are unable to start the problem…. What does average rate of change mean? What is an interval? How can you break up the intervals? Is the trend linear, exponential or quadratic? How many trends do you see?

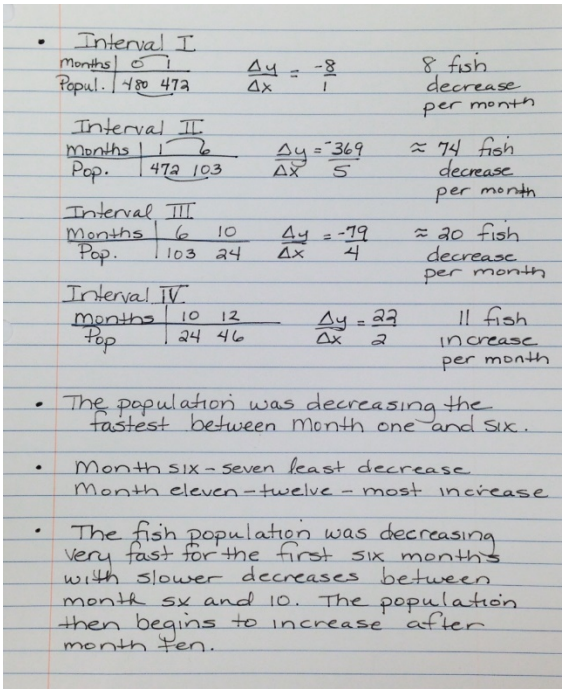
The student has difficulty finding the average rate of change … How do you calculate average rate of change? When is the rate of change positive? On what interval? When is the rate of change negative? On what interval?

The student is having trouble finding intervals … How many intervals are needed? How many trends do you see?

The student is unable to find the population increase or decrease … What is the proper way to represent a percent when multiplying?

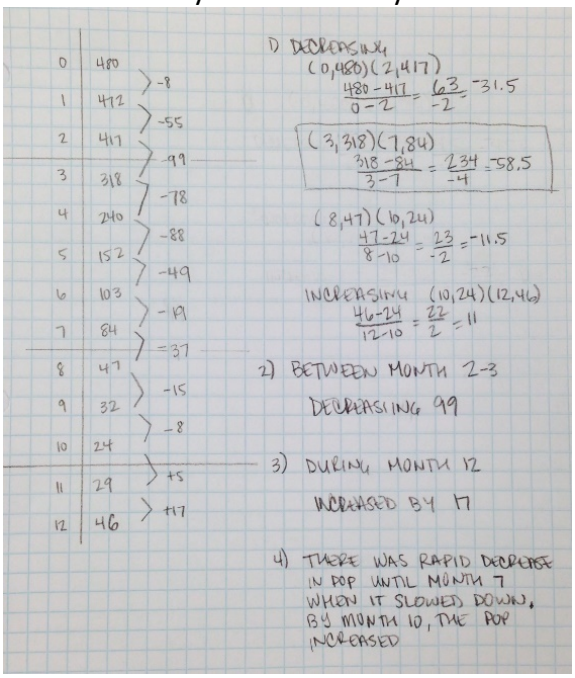
Sample Solutions

Possible Correct Response



- Interval I
  Months | 0 1
  Popul. | 480 472
  $\frac{\Delta y}{\Delta x} = \frac{-8}{1}$
  8 fish decrease per month
- Interval II
  Months | 1 6
  Pop. | 472 103
  $\frac{\Delta y}{\Delta x} = \frac{-369}{5}$
  ≈ 74 fish decrease per month
- Interval III
  Months | 6 10
  Pop. | 103 24
  $\frac{\Delta y}{\Delta x} = \frac{-79}{4}$
  ≈ 20 fish decrease per month
- Interval IV
  Months | 10 12
  Pop | 24 46
  $\frac{\Delta y}{\Delta x} = \frac{22}{2}$
  11 fish increase per month
- The population was decreasing the fastest between Month one and six.
- Month six - seven least decrease
  Month eleven - twelve - most increase
- The fish population was decreasing very fast for the first six months with slower decreases between month six and 10. The population then begins to increase after month ten.

Student responses may vary, justification must be mathematically sound. What kinds of trends do you see? How did you determine your intervals? How do you calculate average rate of change?



0	480	
1	472	-8
2	417	-55
3	318	-99
4	240	-78
5	152	-88
6	103	-49
7	84	-19
8	47	-37
9	32	-15
10	24	-8
11	29	+5
12	46	+17

1) DECREASING
(0,480)(2,417)
$\frac{480-417}{0-2} = \frac{63}{-2} = -31.5$
(3,318)(7,84)
$\frac{318-84}{3-7} = \frac{234}{-4} = -58.5$
(8,47)(10,24)
$\frac{47-24}{8-10} = \frac{23}{-2} = -11.5$
INCREASING (10,24)(12,46)
$\frac{46-24}{12-10} = \frac{22}{2} = 11$

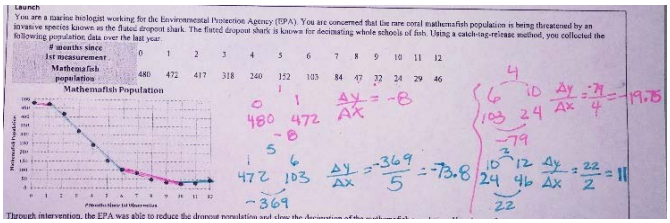
2) BETWEEN MONTH 2-3
DECREASING 99

3) DURING MONTH 12
INCREASED BY 17

4) THERE WAS RAPID DECREASE IN POP UNTIL MONTH 7 WHEN IT SLOWED DOWN. BY MONTH 10, THE POP INCREASED

Student responses may vary, justification must be mathematically sound. What kinds of trends do you see? How did you determine your intervals? How do you calculate average rate of change? How can you explain your solution? Did you explain how you chose your intervals?

Common Incorrect Response



Problem only partially done. Student responses may vary, justification must be mathematically sound. What kinds of trends do you see? How did you determine your intervals? How do you calculate average rate of change? How can you explain your solution?

Debrief

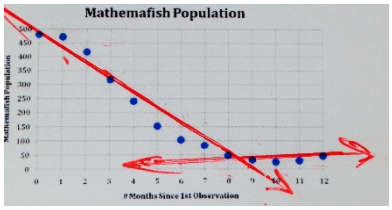
Whole/Large Group Discussion

- Debriefing formats may differ (e.g., whole-class discussion, small-group discussion). It will be beneficial for students to view student work as a gallery walk or similar activity.
- Have students/teacher facilitate the sequence of multiple representations in an order that moves from less to more mathematical sophistication.
- Allow students to question each other and explain their choices, using mathematical reasoning. If students struggle, use questioning strategies.
- Encourage students to notice similarities, differences, and generalizations across strategies.
- Provide constructive feedback and ask clarifying questions for deeper understanding of the process.

If you see this common error…, it might mean this…

Student found slope by inverting the numerator and the denominator. Student created a positive slope when the slope should have been negative… The student may be confused with the slope formula.

Student has too few or too many intervals (without finding a trend)…The student may not understand intervals.



Student is connecting the points… The student may not understand the difference between a continuous and discrete graph.

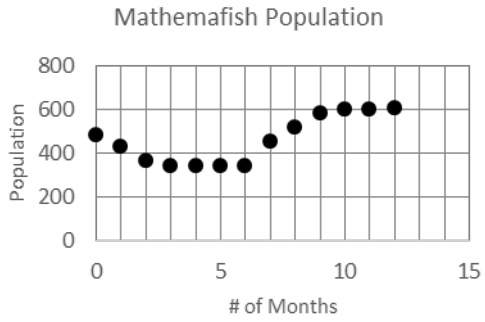
Synthesize and Apply

Monitor student work and facilitate discussions by asking questions. When students have independently arrived at the Enduring Understanding, engage them in solving these extension problems. Assess if you have facilitated the discussion in a way that students have arrived at the Enduring Understanding (do not tell them, they will benefit from discovering it for themselves).

Extension Problem #1

You are a marine biologist working for the Environmental Protection Agency (EPA). You are concerned that the rare coral mathemafish population is being threatened by an invasive species known as the fluted dropout shark. The fluted dropout shark is known for decimating whole schools of fish. Using a catch-tag-release method, you collected the following population data over the last year.

	Mathemafish population
0	480
1	430
2	365
3	342
4	342
5	342
6	342
7	453
8	515
9	583
10	596
11	600
12	605



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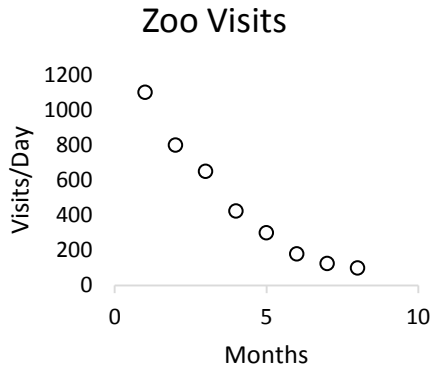
What to include in your summary report:
How many intervals do you see?
Describe the average rate of change for each interval.

During what interval did you notice the largest effect of the EPA intervention? Was it postive or negative?

Extension Problem #2

Look at the graph below. Determine the average decrease in the number of visits per day each month over three different intervals (the intervals can overlap). Explain why you choose those intervals and what the various averages represent in the context of the problem.



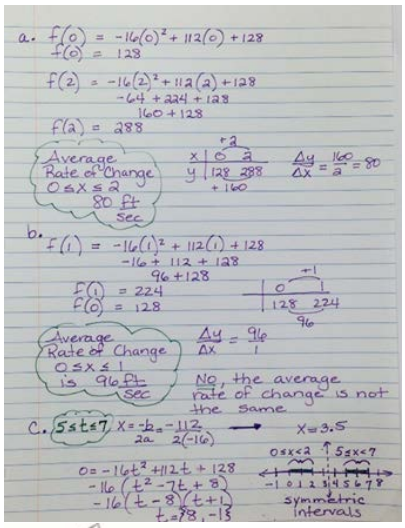
Possible Solutions:

Will vary, students' justifications must be mathematically sound.

Extension Problem #3

The height (in feet) of one of the model rockets launched by a student can be determined using the equation f(t) = -16t2 + 112t + 128, where t is the number of seconds after the rocket is launched.
Part a: What is the average rate of change from zero to two seconds?
Part b: Choose another interval. Is the average rate of change for your interval the same? If yes, will it always be the same? If no, will it ever be the same?
Part c: What interval has the opposite average rate of change as the interval found in Part a?

Possible Solutions:



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Polya, G. (2014). How to solve it: *A new aspect of mathematical method.* Princeton, NJ: Princeton University Press.

Name_____

Student Page

Launch

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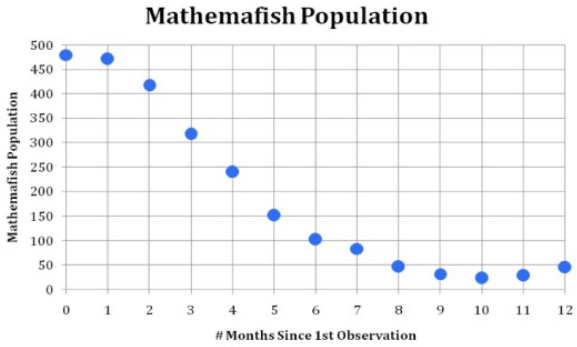

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