

# Buying a Car

## Enduring Understanding

**(Do not tell students; they must discover it for themselves.)**

Students will create and solve equations in one and two variables to represent relationships between quantities.

### Standards

HSA.CED.A.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions and simple rational and exponential functions.

HSA.CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axis with labels and scales.

HSA.CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

HSA.REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

## Launch

### Introduce the Task

Suppose a friend tells you she paid a total of \$16,368 for a car, and you'd like to know the car's list price (the price before taxes) so that you can compare prices at various dealers. Find the list price of the car if your friend bought the car in:

- Arizona, where the sales tax is 6.6%.
- New York, where the sales tax is 8.25%.
- A state where the sales tax is  $r$ .

### Understand the Problem

- Are there any word(s) you don't understand?
- What is the question or task asking you to answer?
- Is there enough information to find a solution?
- Restate the problem in your own words.
- What additional information do you need to find?

## Develop a Plan

- There are many reasonable ways to solve a problem. With practice, students will build the necessary skills to choose an efficient strategy for the given problem.
- Ensure that students have a place to start and that the task/problem has the ability to be scaffolded.
- Caution should be exercised to not force your plan/reasoning on students.

## Investigate

### Productive Struggle

- Let students engage in productive struggle.
- Monitor as students work.
- Offer positive constructive feedback.
- Ask questions such as...
  - Why did you choose that number?
  - What assumptions did you make?
  - Explain what you are doing here.
  - What does that solution mean?

### Questions for Individuals as they Work

**Students are unable to start the problem...** What does the total price of the car consist of? (Where did the total price come from?) What is tax? How can you determine how much your friend paid in taxes? What do they mean by “the list price”? How do you find the list price? Can you write a verbal model? What does your variable represent?

**The student set up the equation incorrectly, for example:  $0.066x = 16,368$  or  $16,368(0.066) = \text{List price}$  or  $6.6x = 16,368$  or  $16,368(6.6) = \text{List price}$  ...** Should the list price be greater or smaller than the total price? Is your answer reasonable? What have you just calculated? What does the total price of the car represent? What does percent mean? How do you calculate with percentages?

**The student is stuck on “ $r$ ” as the percent....** What did you do in part a? Part b? What is the coefficient of  $r$ ? What does  $r$  represent as compared to part a? Part b? How is the wording between part b and part c similar? Can you underline common words?

# Sample Solutions

## Possible Correct Response

### Strategy #1

Verbal Model:

Total is tax plus list price.  
 Total is % of list price plus list price.  
 Let  $x$  represent list price.

$$T = \% \cdot x + x$$

a)  $16,368 = 0.066x + x$   
 $\frac{16,368}{1.066} = \frac{1.066x}{1.066}$   
 $\$15,354.40 = x$

b)  $16,368 = 0.0825x + x$   
 $\frac{16,368}{1.0825} = \frac{1.0825x}{1.0825}$   
 $\$15,120.55 = x$

c)  $16,368 = 0.01r x + x$   
 $\frac{16,368}{1 + 0.01r} = \frac{x(1 + 0.01r)}{(1 + 0.01r)}$   
 $\frac{16,368}{1 + 0.01r} = x$

### Strategy #2

a)  $\frac{X}{100\%} = \frac{16368}{106.6\%}$   
 $\frac{100 \cdot X}{100} = \frac{16368 \cdot 100}{106.6}$   
 $X = \frac{1636800}{106.6} = \$15,354.60$  (Arizona)

b)  $\frac{X}{100\%} = \frac{16368}{108.25}$   
 $\frac{100 \cdot X}{100} = \frac{16368 \cdot 100}{108.25}$   
 $X = \frac{1636800}{108.25} = \$15,120.55$  (New York)

c)  $\frac{X}{100\%} = \frac{16368}{100+r}$   
 $\frac{100 \cdot X}{100} = \frac{16368 \cdot 100}{100+r}$   
 $X = \frac{1636800}{100+r}$

This is one possible correct strategy. How did you get this answer? Are the list prices you got reasonable when compared to the total price of the car? Did you answer each part of the question? Did you check your work? Of the states we compared, in which state would you rather buy a car? Why? What place value did you round to and why?

## Common Incorrect Response

$$\begin{aligned} & b = \text{base (list) price} \\ A) \quad 16,368 &= .066b + b \\ \frac{16,368}{.934} &= \frac{.934b}{.934} \\ 17,524.63 &= b \end{aligned}$$

Mistakes:

- While combining like terms, the student subtracted  $0.066b$  and  $1b$  instead of adding the coefficients.
- The base (list) price is larger than the total price. ( $b$  is too large)

Questions:

- Where is your verbal model?
- What was your given information?
- Does this answer make sense? Why/why not?
- What does  $b$  represent?
- Should the value of  $b$  be larger or smaller than your starting value?
- How did you get  $0.934b$ ?

$$\begin{aligned} A) \quad \text{List price} &= (1 - \%) \text{Total} \\ &= (1 - 0.066) \cdot 16,368 \\ &= (.934) \cdot 16,368 \end{aligned}$$

$$\text{List price} = \$15,287.71$$

Mistakes:

- Students calculated a percent decrease instead of a percent increase (they subtracted the rate when they should have added).
- Students do not understand that tax increases your total price.
- The list price and the total should be reversed.

Questions:

- What would tax do to your list price?
- When calculating the percent of the list price, do you multiply the percent by the total price or the list price?

$$b) \begin{aligned} 16,368 \cdot 8.25 &= x & x = \text{list price} \\ 135,036 &= x \end{aligned}$$

Mistakes:

- 8.25% needs to be converted to a decimal.
- Instead of multiplying the percent with the total value, the total price should be divided by (1 + tax rate).

Questions:

- Where is your verbal model?
- What was your given information?
- Why did you use 8.25?
- Is your answer reasonable for the cost of the car?
- What does  $x$  represent?
- Should the value of  $x$  be larger or smaller than your starting value?

Part c:

$x$  = price of car  
 $r$  = sales tax

$0.0r$  → tax as a decimal

$1.0r$  → total price of car

$$\frac{1.0r}{1.0} = \frac{16,368}{1.0}$$

$r = 16,368$

Mistakes:

- Students moved the decimal on  $r$ , instead of moving the decimal on the coefficient of  $r$ .
- Students solved for  $r$  instead of finding the price of the car.
- The value of the  $r$  is the value of the car instead of a percentage of the list price, which is not correct.

Questions:

- Where is your verbal model?
- What was your given information?
- How did you get  $0.0r$ ?
- Does your equation take into account all the given information?
- Is your equation similar to what you did in part a and part b? Should it be?
- Is your answer reasonable?
- What were you looking for? Did you answer the question that was asked of you?
- Does your answer have to be just a number?



## Debrief

### Whole/Large Group Discussion

- Debriefing formats may differ (e.g., whole-class discussion, small-group discussion). It will be beneficial for students to view student work as a gallery walk or similar activity.
- Have students/teacher facilitate the sequence of multiple representations in an order that moves from less to more mathematical sophistication.
- Allow students to question each other and explain their choices, using mathematical reasoning. If students struggle, use questioning strategies.
- Encourage students to notice similarities, differences, and generalizations across strategies.
- Provide constructive feedback and ask clarifying questions for deeper understanding of the process.

### If you see this common error..., it might mean this...

If a student's answer is:

- Finding 6.6% of \$16,368... Students may not understand "total".
- $x + 0.066x = 0.067x$ ... Students may not understand how to add decimals.
- $0.66x$ ... Students are used to percentages having 2 decimal places, regardless of starting value. (They forgot to move the decimal point to the left twice/They forgot to divide the percent by 100.)
- Not converting the coefficient of  $r$  to a decimal... Students may not understand that  $r$  is a percentage, or a value out of 100. (Students have forgotten to divide  $r$  by 100 somewhere.)
- Part c – Students have divided by  $(1 + r)$  instead of dividing by  $(1 + 0.01r)$ ... If they use  $(1 + r)$ , they will not solve for the list price. Once given  $r$ , they would have to convert it to decimal form before substituting into the formula.
- Part c – Not solving for the list price. Ex:  $16,368 = (1 + 0.01r)b$ , where  $b$  is the list price of the car. Students should have divided by  $(1 + 0.01r)$  to isolate  $b$ ... Students did not understand the question that was asked/Students did not understand what they were supposed to solve for/Students did not finish the problem.

## Synthesize and Apply

Monitor student work and facilitate discussions by asking questions. When students have independently arrived at the Enduring Understanding, engage them in solving these extension problems. Assess if you have facilitated the discussion in a way that students have arrived at the Enduring Understanding (do not tell them, they will benefit from discovering it for themselves).

### Extension Problem #1

Suppose your cousin just bought a new PlayStation 4. He paid a total of \$405.99, and you'd like to know the console's list price (the price before taxes) so that you can compare prices at various stores. Find the list price if your cousin bought the console in:

- Elko, NV, where the sales tax is 6.85%.
- Reno, NV, where the sales tax is 7.725%.
- A city where the sales tax is  $r$ .

### Possible Solution:

Extension Problem #1

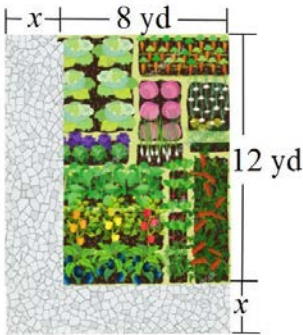
a)  $x = \text{list price}$   
 $405.99 = x + \frac{6.85}{100}x$   
 $405.99 = x + 0.0685x$   
 $\frac{405.99}{1.0685} = \frac{1.0685x}{1.0685}$   
 $379.96 = x$

b)  $x = \text{list price}$   
 $405.99 = x + \frac{7.725}{100}x$   
 $405.99 = x + 0.07725x$   
 $\frac{405.99}{1.07725} = \frac{1.07725x}{1.07725}$   
 $376.88 = x$

c)  $x = \text{list price}$   
 $405.99 = x + \frac{r}{100}x$   
 $\frac{405.99}{(1 + \frac{r}{100})} = \frac{x(1 + \frac{r}{100})}{(1 + \frac{r}{100})}$   
 $\frac{405.99}{1 + \frac{r}{100}} = x$

## Extension Problem #2

You are putting a cobblestone path along two adjacent sides of a rectangular garden that measures 8 yards by 12 yards. You have enough money saved to buy 96 square yards of cobblestone.



(figure not necessarily drawn to scale)

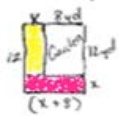
- How wide should the path be?
- If cobblestone costs \$14 per square yard, how much more money would you need to create a path along all four sides of the garden?

### Possible Solution:

Strategy #1:

Solution

a.) Let  $x$  represent the width of the path.

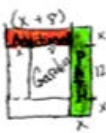


$$\begin{aligned} \text{Area of Rectangular Garden} + \text{Area of Rectangular Path} &= \text{Total Area} \\ l \cdot w + l \cdot w &= A \\ x \cdot 12 + (x+8)x &= 96 \\ 12x + x^2 + 8x &= 96 \\ x^2 + 20x - 96 &= 0 \end{aligned}$$

$$\begin{aligned} x^2 + 20x - 96 &= 0 \\ (x+24)(x-4) &= 0 \\ x = -24 \quad x = 4 \end{aligned}$$

The path is 4 yd wide

b.)



$$\begin{aligned} \text{Area of Rec. Garden} + \text{Area of Rec. Path} &= \text{Total Area of Additional Cobble} \\ l \cdot w + l \cdot w &= A \\ (x+8)x + x(2x+12) &= A \end{aligned}$$

$$\begin{aligned} x^2 + 8x + 2x^2 + 12x &= A \\ 3x^2 + 20x &= A \\ 3(4)^2 + 20(4) &= A \\ 48 + 80 &= A \end{aligned}$$

$$128 = \text{Additional yd}^2$$

Price per yd<sup>2</sup> = \$14  
 Number of Additional yd<sup>2</sup> = 128  
 $14 \cdot 128 \text{ yd}^2 = \$1792$

$x=4$

I need an additional \$1792



Strategy #2:

a)  $12 + 1 + 8 = 21$   
 $13 + 1 + 9 = 23$   
 $14 + 1 + 10 = 25$   
 $15 + 1 + 11 = 27$

The path will be 4 yards wide

b)

$12 + 1 + 16 = 29$   
 $13 + 1 + 17 = 31$   
 $14 + 1 + 18 = 33$   
 $15 + 1 + 19 = 35$

128
.14
1792

You will need \$1,792 to complete the path.

**Extension Problem #3**

Susan wants to make a bowl of candy using Milk Duds, M & M's, and Jelly Beans. She wants to mix one part Milk Duds, three parts M & M's, and four parts Jelly Beans. Milk Duds cost \$8 per pound, M & M's cost \$12 per pound, and Jelly Beans cost \$14 per pound. Let  $x$  represent the number of pounds of Milk Duds.

- a) Susan has spent \$25 on her candy mix. Create and solve an equation to find the number of pounds of Milk Duds Susan will be able to buy.
- b) Determine how many pounds of candy she can put in the candy bowl. Show your work.

**Possible Solution:**

Strategy #1:

Solution Extension Problem #3

1 part Milk Duds  $\rightarrow$  \$8/pound  
 3 parts M+M's  $\rightarrow$  \$12/pound  
 4 parts Jelly Beans  $\rightarrow$  \$14/pound

a)  $x =$  pounds of Milk Duds  
 $3x =$  pounds of M+M's  
 $4x =$  pounds of Jelly Beans

$$8x + 12(3x) + 14(4x) = 25$$

$$8x + 36x + 56x = 25$$

$$100x = 25$$

$$x = .25$$

b)  $3(.25) = .75$  pounds of M+M's  
 $4(.25) = 1$  pound of Jelly Beans

There will be 2 pounds of candy in the bowl.  
 $(.25 + .75 + 1 = 2 \text{ lbs})$



Strategy #1:

3)  $x = 1$  part of the candy bowl

Milk Duds	\$8/pound	= $8x$
M + Ms	\$12/pound	= $12x$
M + Ms		= $12x$
M + Ms		= $12x$
Jelly beans	\$14/pound	= $14x$
Jelly beans		= $14x$
Jelly beans		= $14x$
Jelly beans		= $14x$
\$25 to spend (total)		

A)  $8x + 12x + 12x + 12x + 14x + 14x + 14x + 14x = 25$

$$\frac{100x}{100} = \frac{25}{100}$$

B)

$$x = .25$$

Milk Duds = 1 part =  $x = .25$  pounds  
 M + Ms = 3 parts =  $3x = .75$  pounds  
 Jelly beans = 4 parts =  $4x = 1$  pound  
 total weight = 2 pounds



## References

Common Core State Standards Initiative. (2010). *Common core state standards for mathematics*. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.

### [Illustrative Mathematics](#)

Polya, G. (2014). *How to solve it: A new aspect of mathematical method*. Princeton, NJ: Princeton University Press.

Name \_\_\_\_\_

## Student Page

Suppose a friend tells you she paid a total of \$16,368 for a car, and you'd like to know the car's list price (the price before taxes) so that you can compare prices at various dealers. Find the list price of the car if your friend bought the car in:

- a) Arizona, where the sales tax is 6.6%.
- b) New York, where the sales tax is 8.25%.
- c) A state where the sales tax is  $r$ .