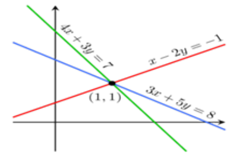


Basketball

Enduring Understanding

(Do not tell students; they must discover it for themselves.)

Students will write and solve equations and inequalities in real-world situations.

Standards

HSA.CED.A.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
HSA.REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
HSF.LE.A.1 Construct and compare linear, quadratic, and exponential models and solve problems.
HSF.LE.A.1.A Distinguish between situations that can be modeled with linear functions and with exponential functions.

Launch

Introduce the Task

Chase and his brother like to play basketball. About a month ago they decided to keep track of how many games they have each won. As of today, Chase has won 18 out of the 30 games against his brother.

a) How many additional games would Chase have to win in a row in order to have a 75% winning record?
b) How many additional games would Chase have to win in a row in order to have a 90% winning record?
c) Is Chase able to reach a 100% winning record? Explain why or why not.
d) Suppose that after reaching a winning record of 90% in part (b), Chase had a losing streak. How many additional games in a row would Chase have to lose in order to drop down to a winning record below 55% again?

Understand the Problem

- Are there any word(s) you don't understand?
- What is the question or task asking you to answer?
- Is there enough information to find a solution?
- Restate the problem in your own words.
- What additional information do you need to find?

Develop a Plan

- There are many reasonable ways to solve a problem. With practice, students will build the necessary skills to choose an efficient strategy for the given problem.
- Ensure that students have a place to start and that the task/problem has the ability to be scaffolded.
- Caution should be exercised to not force your plan/reasoning on students.

Investigate

Productive Struggle

- Let students engage in productive struggle.
- Monitor as students work.
- Offer positive constructive feedback.
- Ask questions such as…
    - Why did you choose that number?
    - What assumptions did you make?
    - Explain what you are doing here.
    - What does that solution mean?

Questions for Individuals as they Work

Students are unable to start the problem…. How can you represent 18 out of 30? What are you trying to find? What variables did you define?

The student does not include playing more games… How will you show the siblings playing more games? What does your variable represent? What happens if Chase wins the next game?

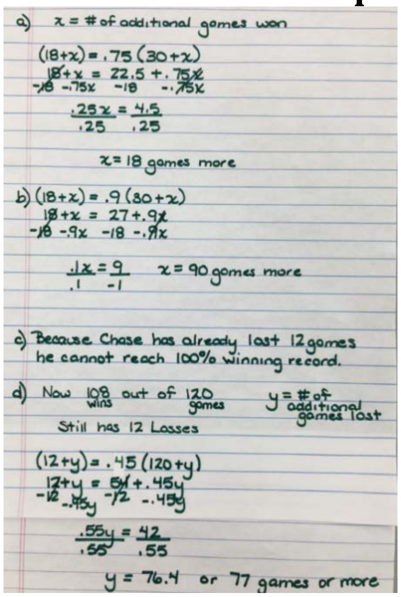
The student does not set up a proportion… How can you set up an equation to help you solve this problem? What do the parts of your equation represent?

The student does not write the percent as a decimal… What is the proper way to represent a percent when multiplying?

The student does not define a variable correctly… What is the unknown? What does your variable represent?

Sample Solutions

Possible Correct Response



a) x = # of additional games won

$(18+x) = .75(30+x)$

$18+x = 22.5+.75x$

$-18 \; -.75x \quad -18 \; -.75x$

$\frac{.25x}{.25} = \frac{4.5}{.25}$

$x = 18$ games more

b) $(18+x) = .9(30+x)$

$18+x = 27+.9x$

$-18 \; -.9x \quad -18 \; -.9x$

$\frac{.1x}{.1} = \frac{9}{.1}$ $x = 90$ games more

c) Because Chase has already lost 12 games he cannot reach 100% winning record.

d) Now 108 wins out of 120 games y = # of additional games lost

Still has 12 Losses

$(12+y) = .45(120+y)$

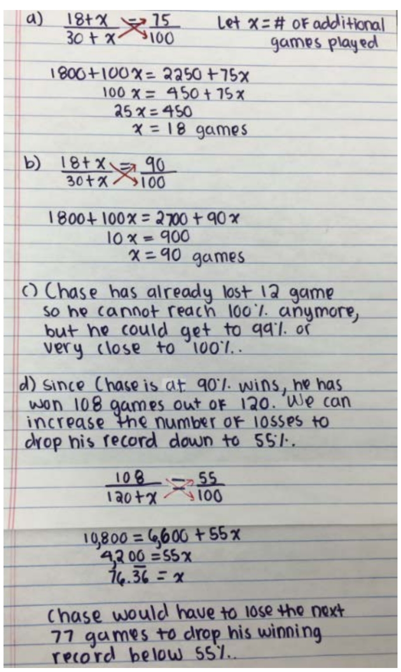
$12+y = 54+.45y$

$-12 \; -.45y \quad -12 \; -.45y$

$\frac{.55y}{.55} = \frac{42}{.55}$

$y = 76.4$ or 77 games or more

What does your variable represent? Why did you set up your equation this way? What does 100% actually mean in this context in part c? Why did you introduce a second variable in part d? Could part d have been written as an inequality rather than an equation?



a) $\frac{18+x}{30+x} = \frac{75}{100}$ Let x = # of additional games played

$1800+100x = 2250+75x$

$100x = 450+75x$

$25x = 450$

$x = 18$ games

b) $\frac{18+x}{30+x} = \frac{90}{100}$

$1800+100x = 2700+90x$

$10x = 900$

$x = 90$ games

c) Chase has already lost 12 game so he cannot reach 100% anymore, but he could get to 99% or very close to 100%.

d) since Chase is at 90% wins, he has won 108 games out of 120. We can increase the number of losses to drop his record down to 55%.

$\frac{108}{120+x} = \frac{55}{100}$

$10{,}800 = 6{,}600+55x$

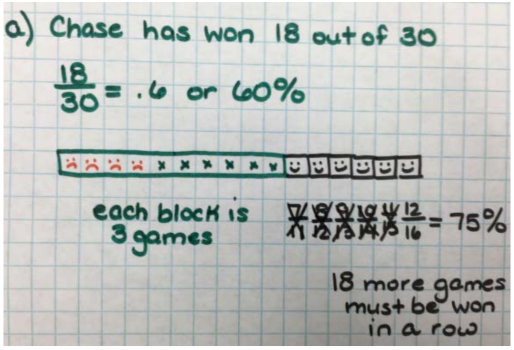
$4200 = 55x$

$76.\overline{36} = x$

Chase would have to lose the next 77 games to drop his winning record below 55%.

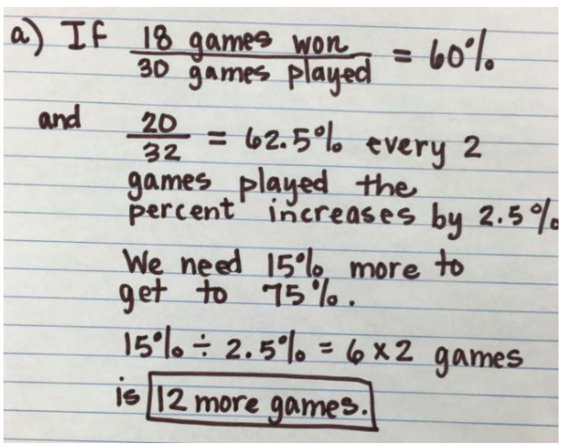
Why did you use proportions? What does your variable represent? In part c, what is the closest you can get to 100%? Could part d have been written as an inequality rather than an equation? In part d, why did you decide to look at the total number of games rather than the total number of losses?

Tape Diagram – Alternative Strategy



How did you determine that every block equals 3 games? How did you get to 75%? What do you notice when you increase the tape diagram by another block (which equals 3 games)? Why did you stop at 12/16?

Common Incorrect Response



Have you tried checking other ratios to verify your claim that every 2 games played increases the percent by 2.5%? Could you use a table to help organize your information?

Debrief

Whole/Large Group Discussion

- Debriefing formats may differ (e.g., whole-class discussion, small-group discussion). It will be beneficial for students to view student work as a gallery walk or similar activity.
- Have students/teacher facilitate the sequence of multiple representations in an order that moves from less to more mathematical sophistication.
- Allow students to question each other and explain their choices, using mathematical reasoning. If students struggle, use questioning strategies.
- Encourage students to notice similarities, differences, and generalizations across strategies.
- Provide constructive feedback and ask clarifying questions for deeper understanding of the process.

If you see this common error…, it might mean this…

Students increase the number of games won but not the number of total games… The student does not understand how an additional game will affect an equation or proportion.

Students cannot convert from percents to either decimals or fractions… The student does not understand the meaning of percent.

Students are unsure of how to solve a proportion… The student does not understand what two equivalent expressions represent and that the properties of equality can be used.

Synthesize and Apply

Monitor student work and facilitate discussions by asking questions. When students have independently arrived at the Enduring Understanding, engage them in solving these extension problems. Assess if you have facilitated the discussion in a way that students have arrived at the Enduring Understanding (do not tell them, they will benefit from discovering it for themselves).

Extension Problem #1

Linda and her friend are comparing their grades in class. They decide to keep track of the amount of points they've accumulated. As of today, Linda has 195 points out of 250 points.

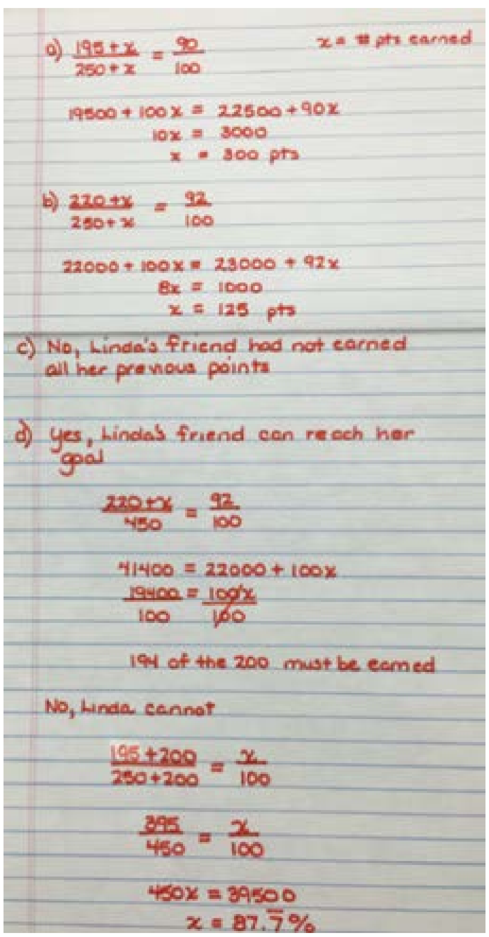
a) How many additional consecutive points does Linda need to earn to get her grade to 90%?

b) Linda's friend has 220 points out of 250, how many points does her friend need to obtain to get a grade of 92%?

c) Linda thinks her friend can get to 100%. Is she correct?

d) If only 200 points are available for the rest of the grading period, can Linda and her friend reach their goals of 90% and 92%?

Possible Solution:



a) $\frac{195+x}{250+x} = \frac{90}{100}$ x = # pts earned

$19500 + 100x = 22500 + 90x$

$10x = 3000$

$x = 300$ pts

b) $\frac{220+x}{250+x} = \frac{92}{100}$

$22000 + 100x = 23000 + 92x$

$8x = 1000$

$x = 125$ pts

c) No, Linda's friend had not earned all her previous points

d) Yes, Linda's friend can reach her goal

$\frac{220+x}{450} = \frac{92}{100}$

$41400 = 22000 + 100x$

$\frac{19400}{100} = \frac{100x}{100}$

194 of the 200 must be earned

No, Linda cannot

$\frac{195+200}{250+200} = \frac{x}{100}$

$\frac{395}{450} = \frac{x}{100}$

$450x = 39500$

$x = 87.\overline{7}\%$

Extension Problem #2

You have decided to open a checking account at the Bank of George. Bank of George is offering you a free checking account if you maintain a minimum balance of $200. You already have a savings account with this bank and you have $60 saved. You decide to keep saving money until you have enough to open a checking account, plus keep some money in savings. You can afford to deposit $15 a week into the savings account and wish to open a free checking account while maintaining $25 in the savings account.

a) What is the minimum amount of money you want to have to open your checking account?

b) What inequality can be written to model the scenario?

c) What is the solution to the inequality?

d) What does the solution of the inequality represent in terms of the context of the problem?

Possible Solution:

a) $225

b) What inequality can be written to model the scenario?

$$60 + 15x \geq 225$$

c) What is the solution to the inequality?

$$x \geq 11$$

d) It will take at least 11 weeks to reach the savings goal of having $200 to open the checking account while still having $25 in the savings account.

Extension Problem #3

On opposite sides of a major city, two suburban towns are experiencing population changes. One town, Town A, is growing rapidly at 5% per year and has a current population of 39,000. Town B has a declining population at a rate of 2% per year. Its current population is 55,000. Economists predict that in 5 years the populations of these two towns will be about the same, but the residents of both towns are in disbelief. The economists also claim that ten years after that, Town A will double the size of Town B. Can you verify the predictions based on the data given? Do you think these predictions will come true?

a) Find the equations for Town A and Town B populations and then find the population values for each town after 5 years.

b) Are the populations similar in Town A and Town B after 5 years? What can you conclude about the economists' prediction?

c) Find the populations for Town A and Town B ten years after these new figures.

d) Was the economists' prediction for the town populations after 10 more years correct? What factors might influence whether or not the economists' predictions come true?

Possible Solution:

a) Town A: $y = 39{,}000(1 + 0.05)^t$

Town B: $y = 55{,}000(1 - 0.02)^t$

Town A approximately 49,775 people

Town B approximately 49,716 people

b) The populations are very similar and it seems the economists are correct that the populations will be about equal in 5 years.

c) After 10 more years, Town A is approximately 81,079 people and Town B is approximately 40,622 people.

d) Yes, the economists' prediction is correct as 81,079 is about twice as much as 40,622. Many factors might contribute to the predictions coming true or not, such as the economy itself. If the economy starts to decline, the population of Town A might not continue to grow as fast, or a new industry might relocate to Town B, bringing jobs and perhaps people will start moving back into that town. Town A might reach its capacity before it reaches 81,079.

References

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Polya, G. (2014). How to solve it: *A new aspect of mathematical method.* Princeton, NJ: Princeton University Press.

Name______________________________

Student Page

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