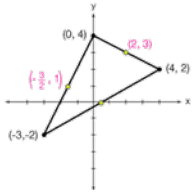


Squares on a Coordinate Grid

Enduring Understanding

(Do not tell students; they must discover it for themselves.)

Students will use the coordinate system and/or algebraic methods to verify properties of special parallelograms. Students will find the area of a polygon in the coordinate plane.

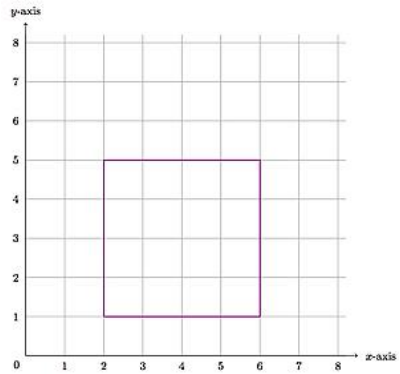
Standards

This task might address the following standards (standards might vary based on discussion)
HSG-GPE.B.4 Use **coordinates to prove simple geometric theorems algebraically** Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).*
HSG-GPE.B.7 Use **coordinates to prove simple geometric theorems algebraically** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*
HSG-CO.C.11 Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

Launch

Introduce the Task

In the picture below a square is outlined whose vertices lie on the coordinate grid points:



The square on the grid has an area of 16 square units. Find a square with vertices on the coordinate grid whose area is *n* square units, where *n* is a whole number between 1 and 10, or show that there is no such square.

Understand the Problem

- Are there any word(s) you don't understand?
- What is the question or task asking you to answer?
- Is there enough information to find a solution?
- Restate the problem in your own words.
- What additional information do you need to find?

Develop a Plan

- There are many reasonable ways to solve a problem. With practice, students will build the necessary skills to choose an efficient strategy for the given problem.
- Ensure that students have a place to start and that the task/problem has the ability to be scaffolded.
- Caution should be exercised to not force your plan/reasoning on students.

Investigate

Productive Struggle

- Let students engage in productive struggle.
- Monitor as students work.
- Offer positive constructive feedback.
- Ask questions such as…
    - Why did you choose that number?
    - What assumptions did you make?
    - Explain what you are doing here.
    - What does that solution mean?

Questions for Individuals as they Work

Students misunderstanding the question…

What does *n* represent in the problem?

What is a vertex?

What does it mean that the vertices (points) are on the coordinate grid?

Students don't know how to find the area of a square…

How do you find the area of a square?

Is the process to find the area of a rectangle the same as the area of a square?

Does the square have special properties?

What are those special properties?

Students using a quadrant other than quadrant 1…

What formulas did you use to find your answer?

Make sure that students are correctly calculating the side lengths.

Students drawing non-square shapes…

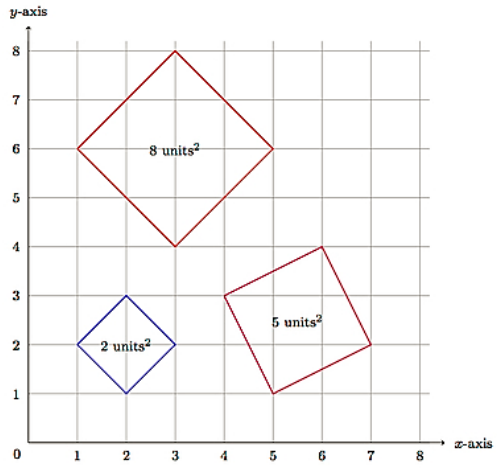
What are the properties of the shape you drew?

What does the shape of a square look like?

What are the special properties of a square?

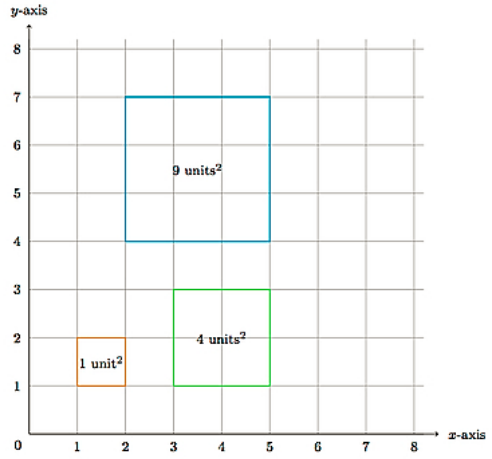
Sample Solutions

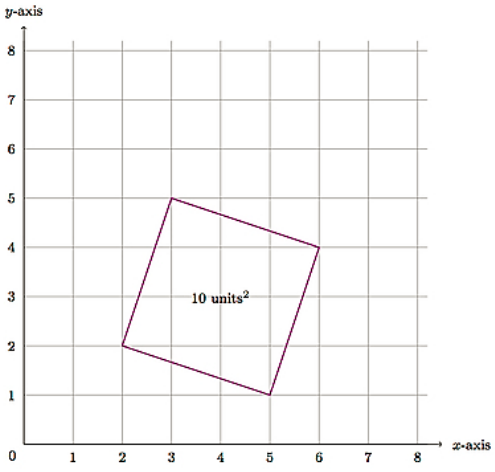
Possible correct response:



- Are these the only possible solutions?
- Can you find areas of squares whose sides are not parallel to the axis?
- Do you need to find all side lengths or are there some short cuts?
- What are the special properties of squares?

- How do you know if those are squares?
- What was your process in finding the sides of a square that are not parallel to the *x* or *y*-axis?
- Is there an algebraic process to help determine your answer?
- How did you use Pythagorean's Theorem to help determine which number units would not work?
- How would you use the slope formula to determine the relationships between the opposite sides of a polygon?
- How would you use the slope formula to determine the relationships between the adjacent sides of a polygon?
- When trying to draw the image of the unit squares that did not work, what did you determine about the vertices lying on the coordinate plane?
- Is it possible to put the vertices on the coordinate grid?





There are no squares of area 3, 6, and 7 units^2 whose vertices lie at grid points.

Debrief

Whole/Large Group Discussion

- Debriefing formats may differ (e.g., whole-class discussion, small-group discussion). It will be beneficial for students to view student work as a gallery walk or similar activity.
- Have students/teacher facilitate the sequence of multiple representations in an order that moves from less to more mathematical sophistication.
- Allow students to question each other and explain their choices, using mathematical reasoning. If students struggle, use questioning strategies.
- Encourage students to notice similarities, differences, and generalizations across strategies.
- Provide constructive feedback and ask clarifying questions for deeper understanding of the process.

If you observe this …, you might ask this ….

Students finding the squares that are parallel to the *x* and *y*-axis. (This limits the number of possible answers.)…What areas did you find between 1 and 10? Do the sides have to be parallel to the axis? Look to see if you can find areas of squares with sides that are not parallel to the axis. Do the side lengths have to be whole numbers?
Students stop at one example…. Are there more possible solutions? Can you draw and identify their areas?

Students get squares outside the parameter (too large).. What are the restrictions on the area of the square?

If you see this common error…, it might mean this…

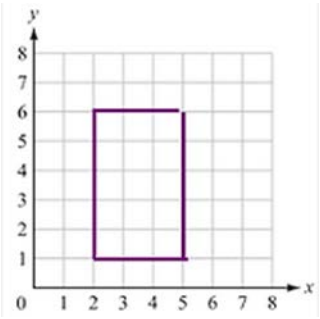
Students only find solutions only for perfect squares 1, 4, and 9. Are there sides that may not be a perfect square? Try to see if the area of 2 might be possible? Try using a diagonal line.
Students do not calculate the side length correctly. Have you checked your formula/calculations? Is there another way to calculate the side length (Pythagorean Theorem vs Distance Formula)?

Synthesize and Apply

Monitor student work and facilitate discussions by asking questions. When students have independently arrived at the Enduring Understanding, engage them in solving these extension problems. Assess if you have facilitated the discussion in a way that students have arrived at the Enduring Understanding (do not tell them, they will benefit from discovering it for themselves).

Extension Problem #1

In the picture below a rectangle is outlined whose vertices lie on the coordinate grid points:

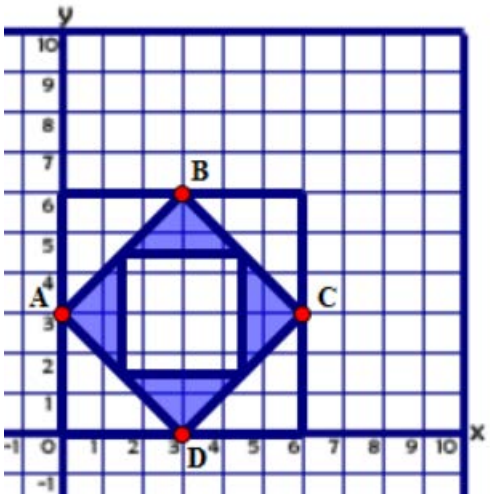


The area of this particular rectangle is 15 square units. For each whole number n between 1 and 10, find a rectangle with vertices on the coordinate grid whose area is n square units or show that there is no such rectangle.

Possible Solutions:

1x1, 1x2, 1x3, 2x2, 1x5, 2x3, 1x7,2x4, 3x3, 2x5

Extension Problem #2

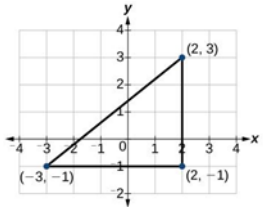


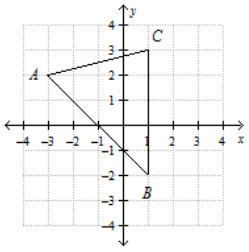
Prove that ABCD is a square. What is the area of the blue-shaded material? Explain.

Possible Solutions:

Proofs may be paragraph or two-column. Opposite sides are parallel and consecutive sides are congruent therefore, ABCD is a rhombus. One of the angles is a right angle therefore all angles are right angles by properties of a parallelogram, therefore ABCD is a rectangle. Since ABCD is both a rectangle and a rhombus, then ABCD is a square.
The area of the blue shaded material is 8 square inches if the vertices of the inside square are midpoints of the sides of the square ABCD.
Each triangle has an area of 2. There are 4 triangles; therefore the area of the blue shaded area of 8 square inches. Another way to look at it is to see that two of the blue triangles make a square. Each side of the square is 2 units. Each square is 4 units. There are two squares with the total area of 8 square inches.

Extension Problem #3





Compare the areas of the two triangles. For each whole number *n* from 1 to 9, find a non-right triangle with vertices on the coordinate grid whose area is *n* square units or show that there is no such triangle.

Possible Solutions:

2 – 9 are all possible (Note: 1 can make two different right triangles)

Reminder ($A_T = \frac{1}{2}bh$), base can be any side. Height is perpendicular to base.

References

Common Core State Standards Initiative. (2010). *Common core state standards for mathematics.* Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.

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Polya, G. (2014). How to solve it: *A new aspect of mathematical method.* Princeton, NJ: Princeton University Press.