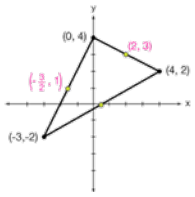


Symmetries of Rectangles

Enduring Understanding

(Do not tell students; they must discover it for themselves.)

Students will be able to recognize lines of symmetry for polygons and define rigid rotations that carry regular polygons onto themselves.

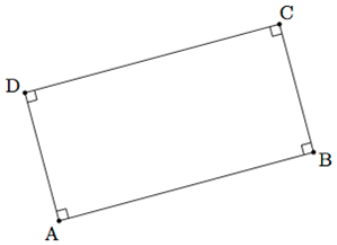
Standards

G-CO.A.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

Launch

Introduce the Task

Jennifer draws the rectangle *ABCD* below:



a) Find all rotations and reflections that will carry rectangle *ABCD* onto itself.
b) Lisa draws a different rectangle and she finds a larger number of symmetries (than Jennifer) for her rectangle. What can you conclude about Lisa's rectangle? Explain.

Understand the Problem

- Are there any word(s) you don't understand?
- What is the question or task asking you to answer?
- Is there enough information to find a solution?
- Restate the problem in your own words.
- What additional information do you need to find?

Develop a Plan

- There are many reasonable ways to solve a problem. With practice, students will build the necessary skills to choose an efficient strategy for the given problem.
- Ensure that students have a place to start and that the task/problem has the ability to be scaffolded.
- Caution should be exercised to not force your plan/reasoning on students.

Investigate

Productive Struggle

- Let students engage in productive struggle.
- Monitor as students work.
- Offer positive constructive feedback.
- Ask questions such as…
    - Why did you choose that number?
    - What assumptions did you make?
    - Explain what you are doing here.
    - What does that solution mean?

Questions for Individuals as they Work

If students are unable to find reflection lines……, then ask:

What does symmetry mean?

How many lines of symmetry can you find?

If students are having a hard time starting the question..., then ask:

Have you identified and sketched each of the quadrilaterals listed?

If students are having problems sketching a line of symmetry..., then ask:

Have you drawn the shape on patty paper and attempted to fold along the line of symmetry?

If students can draw the lines but do not describe the line in words...., then ask:

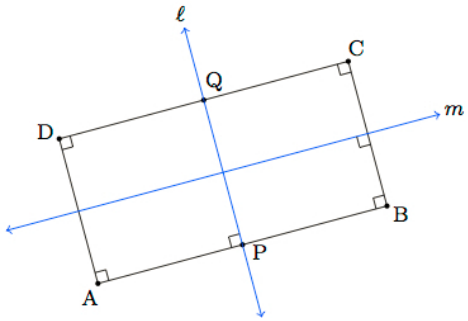
What relationship does the line you've drawn have with existing segments?

Sample Solutions

Possible Solution:

a) Create the perpendicular bisector of $\overline{DC}$, labeled *l,* and reflect *ABCD* over line *l*. Create the perpendicular bisector of $\overline{DA}$, labeled *m,* and reflect *ABCD* over line *m*. Label the intersection of *m* and *l* as point *W*. *ABCD* can be rotated 180° and 360° clockwise or counter-clockwise about *W*.

b) Lisa's rectangle must be a square.



Debrief

Whole/Large Group Discussion

- Debriefing formats may differ (e.g., whole-class discussion, small-group discussion). It will be beneficial for students to view student work as a gallery walk or similar activity.
- Have students/teacher facilitate the sequence of multiple representations in an order that moves from less to more mathematical sophistication.
- Allow students to question each other and explain their choices, using mathematical reasoning. If students struggle, use questioning strategies.
- Encourage students to notice similarities, differences, and generalizations across strategies.
- Provide constructive feedback and ask clarifying questions for deeper understanding of the process.

If you observe this …, you might ask this ….

Students only see rotational symmetry at 180°…Remind students: when the figure rotates back to the starting point, that is considered rotational symmetry also.

Remind students: when the figure rotates back to the starting point, that is considered rotational symmetry also. How many rotations and reflections did the rectangle have? Have you experimented with other types of quadrilaterals? Are any of those also considered rectangles?

Students include diagonals as rotational symmetry. Can you illustrate the reflections across the diagonals? If you fold the rectangle on these lines will it match up? Try with patty paper.

Synthesize and Apply

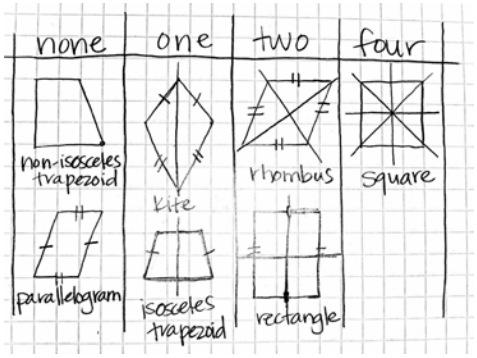
Monitor student work and facilitate discussions by asking questions. When students have independently arrived at the Enduring Understanding, engage them in solving these extension problems. Assess if you have facilitated the discussion in a way that students have arrived at the Enduring Understanding (do not tell them, they will benefit from discovering it for themselves).

Extension Problem #1

Classify the following quadrilaterals based on the number of lines of symmetry that they have. Give a sketch.

- 1) Square
- 2) Rectangle
- 3) Kite
- 4) Isosceles Trapezoid
- 5) Parallelogram
- 6) Rhombus
- 7) Non-Isosceles Trapezoid

Possible Solution:



Extension Problem #2

(Provide graph paper)

A quadrilateral *DEFG* has vertices *D*(-3, 0), *E*(0, 5), *F*(3, 0) and *G*(0, -5). Describe all the reflections and rotations that will map the quadrilateral onto itself.

Possible Solution:

A reflection across the *x* and *y*-axis

Rotations of 180° and 360° about the origin.

Extension Problem #3

A regular *n*-gon is a polygon with *n* sides where all the sides and angles are congruent. For example when *n* = 3 the *n*-gon is an equilateral triangle, when *n* = 6 the *n*-gon is a regular hexagon.

a) How many lines of symmetry does a regular *n*-gon have?

b) What is the angle of a rotational symmetry for a regular *n*-gon.

Possible Solution:

a) *n* lines of symmetry.

b) $\frac{360}{n}^{\circ}$ for the angle of rotational symmetry.

References

Common Core State Standards Initiative. (2010). *Common core state standards for mathematics.* Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.

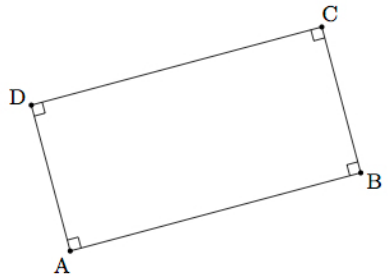
Illustrative Mathematics

Polya, G. (2014). How to solve it: *A new aspect of mathematical method.* Princeton, NJ: Princeton University Press.

Name_____

Student Page

Jennifer draws the rectangle *ABCD* below:



a) Find all rotations and reflections that carry rectangle *ABCD* onto itself.

b) Lisa draws a different rectangle and she finds a larger number of symmetries (than Jennifer) for her rectangle. What can you conclude about Lisa's rectangle? Explain.