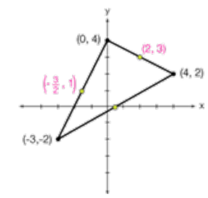
EOC Math II Remediation Lessons

EOC II with a Focus on Geometry



Probability of Rain

Enduring Understanding

(Do not tell students; they must discover it for themselves.)

Students will use product of probabilities to determine independence; calculate intersection of dependent events using appropriate formula; calculate union of events using general addition rule.

Standards

HSS-CP.A.2 **Understand independence and conditional probability and use them to interpret data** Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

HSS-CP.A.3 **Understand independence and conditional probability and use them to interpret data** Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.

HSS-CP.B.6 **Use the rules of probability to compute probabilities of compound events in a uniform probability model** Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.

HSS-CP.B.7 **Use the rules of probability to compute probabilities of compound events in a uniform probability model** Apply the Addition Rule, *P*(*A* or *B*) = *P*(*A*) + *P*(*B*) – *P*(*A* and *B*), and interpret the answer in terms of the model.

HSS-CP.B.8 **Use the rules of probability to compute probabilities of compound events in a uniform probability model** (+) Apply the general Multiplication Rule in a uniform probability model, *P*(*A* and *B*) = *P*(*A*)·*P*(*B*|*A*) = *P*(*B*)·*P*(*A*|*B*), and interpret the answer in terms of the model.

Launch

Introduce the Task

a) Today there is a 55% chance of rain, a 20% chance of lightning, and a 15% chance of lightning and rain together. Are the two events "rain today" and "lightning today" independent events? Justify your answer.

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b) Now suppose that today there is a 60% chance of rain, a 15% chance of lightning, and a 20% chance of lightning if it's raining. What is the chance of both rain and lightning today?

c) Now suppose that today there is a 55% chance of rain, a 20% chance of lightning, and a 15% chance of lightning and rain. What is the chance that we will have rain or lightning today?

d) Now suppose that today there is a 50% chance of rain, a 60% chance of rain or lightning, and a 15% chance of rain and lightning. What is the chance that we will have lightning today?

Debrief

Understand the Problem

- Are there any word(s) you don't understand?
- What is the question or task asking you to answer?
- Is there enough information to find a solution?
- Restate the problem in your own words.
- What additional information do you need to find?

Develop a Plan

- There are many reasonable ways to solve a problem. With practice, students will build the necessary skills to choose an efficient strategy for the given problem.
- Ensure that students have a place to start and that the task/problem has the ability to be scaffolded.
- Caution should be exercised to not force your plan/reasoning on students.

Investigate

Productive Struggle

- Let students engage in productive struggle.
- Monitor as students work.
- Offer positive constructive feedback.
- Ask questions such as…
    - Why did you choose that number?
    - What assumptions did you make?
    - Explain what you are doing here.
    - What does that solution mean?

Questions for Individuals as they Work

Student has difficulty starting the problem, confusion after reading the problem, or trouble starting parts b and c… What is the difference between intersection and union? What keywords define intersection and union? ("and" and "or") How can you visualize or illustrate the problem? (Venn diagram or frequency table)

Student does not remember how to calculate independence in part a.......

Do you know what the formula $P(B) \cdot P(A) = P(B \text{ and } A)$ means?

Explain that when A and B are independent then $P(B) \cdot P(A)$ should be the same as $P(B \text{ and } A)$.

Student does not remember how to calculate independence in part b.......

Can you identify the conditional probability notation? What values do you need in this formula to calculate the probability in this problem? Formula is $P(A \text{ and } B) = P(A) \cdot P(B|A)$

Students are unsure of what formula to use for part c & d…

Note: Students have to use the Addition Rule: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

Sample Solutions

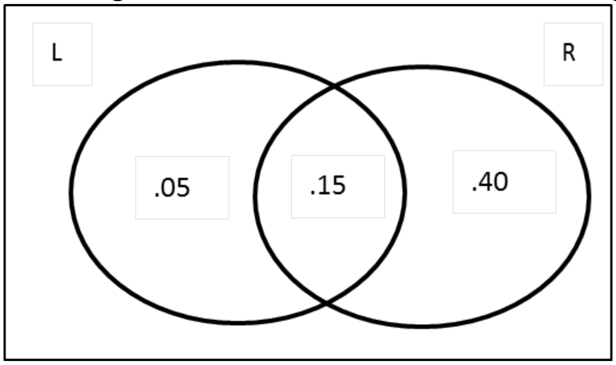
Possible correct response:

a) Test for Independence, formula; $P(A) \cdot P(B) = P(A \text{ and } B)$
$P(A) = P(R) = 0.55$, $P(B) = P(L) = 0.2$,
$P(A \text{ and } B) = P(R \text{ and } L) = 0.15$
$P(R) \cdot P(L) = (0.55) \cdot (0.2) = 0.11$, $P(R \text{ and } L) = 0.15$
Since $0.11 \neq 0.15$, the events are not independent.
b) Conditional Probability formula; $P(A \text{ and } B) = P(A) \cdot P(B|A)$
$P(R) = 0.6$, $P(L) = 0.15$, $P(B|A) = P(L|R) = 0.2$, $P(L \text{ and } R) = ?$
$P(L \text{ and } R) = P(R) \cdot P(L|R) = (0.6)(0.2) = 0.12$
There is a 12% chance of both rain and lightning today.

Can you rearrange the formula to solve explicitly for $P(L|R)$? (or they can substitute first and isolate $P(L|R)$)

c) Method I - Formula
Addition Rule formula; $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
$P(R) = 0.55$, $P(L) = 0.2$, $P(L \text{ and } R) = 0.15$, $P(R \text{ or } L) = ?$
$P(R \text{ or } L) = P(R) + P(L) - P(R \text{ and } L) = 0.55 + 0.2 - 0.15 = 0.6$
There is a 60% chance of rain or lighting today.
Can you use the sum of the components to find the probability of L or R?

Method II - Venn Diagram
Use the given information to create a Venn diagram.



d) Addition Rule formula; $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$P(R) = 0.50$, $P(L) = ?$, $P(L \text{ and } R) = 0.15$

$P(R \text{ or } L) = 0.6$ {from part c}, $P(R \text{ or } L) = P(R) + P(L) - P(R \text{ and } L)$

$0.6 = 0.5 + P(L) - 0.15$, $0.6 = 0.35 + P(L)$, $P(L) = 0.6 - 0.35 = 0.25$

The chance that there will be lightning today $P(L)$ is 25 %.

Debrief

Whole/Large Group Discussion

- Debriefing formats may differ (e.g., whole-class discussion, small-group discussion). It will be beneficial for students to view student work as a gallery walk or similar activity.
- Have students/teacher facilitate the sequence of multiple representations in an order that moves from less to more mathematical sophistication.
- Allow students to question each other and explain their choices, using mathematical reasoning. If students struggle, use questioning strategies.
- Encourage students to notice similarities, differences, and generalizations across strategies.
- Provide constructive feedback and ask clarifying questions for deeper understanding of the process.

If you observe this …, you might ask this ….

Students having algebraic errors… Did you follow the order of operations?

If you see this common error…, it might mean this…

Students using whole numbers as opposed to decimals **…** Did you convert the percentages to decimals?

Students unsure of meaning of the "if" in the statement "20% chance of lighting if it's raining."… What does "if it's raining" mean? (If associates a condition to a statement. If creates a conditional statement.)

Synthesize and Apply

Monitor student work and facilitate discussions by asking questions. When students have independently arrived at the Enduring Understanding, engage them in solving these extension problems. Assess if you have facilitated the discussion in a way that students have arrived at the Enduring Understanding (do not tell them, they will benefit from discovering it for themselves).

Extension Problem #1

a) The probability of having high cholesterol is 32%, the probability of high blood pressure 37%, and the probability of having both high blood pressure and high cholesterol is 11%. Determine if having high blood pressure and having high cholesterol are independent or dependent events? Justify your answers.
b) The probability of high blood pressure is 40%, the probability of high cholesterol 55%, and the probability of high cholesterol if one has high blood pressure is 30%. What is the probability of high blood pressure and high cholesterol?

Possible Solutions:

a) $P(A \text{ and } B) = P(A)\cdot P(B|A)$ must be true for independence.
$P(A) = P(HC) = 0.32$
$P(B) = P(HBP) = 0.37$
$P(A \text{ and } B) = P(HC \text{ and } HBP) = 0.11$
$P(HC)\cdot P(HBP) = (0.32)(0.37) = 0.1184$
$P(HC)\cdot P(HBP) = 0.1184 \neq 0.11$
Since $P(HC)\cdot P(HBP)$ is not equal to $P(HC \text{ and } HBP)$, the events are dependent.

b) $P(A \text{ and } B) = P(A)\cdot P(B|A)$
$P(A \text{ and } B) = ?$
$P(\text{A}) = P(HBP) = 0.40$
$P(B|A) = P(HC|HBP) = 0.30$
Substitute into the formula and solve:
$P(A \text{ and } B) = P(A)\cdot P(B|A)$
$P(HBP \text{ and } HC) = P(HBP)\cdot P(HC|HBP)$
$P(HBP \text{ and } HC) = (0.4)(0.03) = 0.12$
The probability of high blood pressure and high cholesterol is 12%.

Extension Problem #2

64% of students have a twitter account and 28% of students have a Instagram 22% of students have an Instagram and a twitter.

a) Create a two-way table to find the probability that they have no Instagram and no twitter accounts?

b) What is the probability a student has an Instagram or a twitter?

Possible Solutions:

a) P(Twitter) = 0.64

P(Instagram) = 0.28

P(Twitter and Instagram) = 0.22

		Twitter		
		Y	N	
Insta-gram	Y	0.22	0.06	0.28
	N	0.42	0.30	0.72
		0.64	0.36	1.00

Probability of students having no Instagram and no twitter is 30%.

b) Formula:

P(A or B) = P(A) + P(B) – P(A and B)

P(I or T) = P(I) + P(T) – P(I and T)

P(I) = 0.64

P(T) = 0.36

P(I and T) = 0.22

P(I or T) = 0.64 + 0.28 - 0.22 = 0.7

The probability that a student has an Instagram or a Twitter account is 70%.

Extension Problem #3

One card is selected at random from the following set of 6 cards, each of which has a number and a black or white symbol:

{2Δ , 4□, 8■, 8 ◊, 5□, 5■}

a). Let B be the event that the selected card has a black symbol, and F be the event that the selected card has a 5.

b). Are the events B and F independent? Justify your answer with appropriate calculations.

Possible Solutions:

a) Two events A and B are independent if

$P(A)\cdot P(B) = P(A \text{ and } B)$

B and F are independent if $P(B)\cdot P(F) = P(B \text{ and } F)$

$P(B) = \frac{3}{6} = \frac{1}{2}$

$P(F) = \frac{2}{6} = \frac{1}{3}$

$P(B \text{ and } F) = \frac{1}{6}$

$P(B)\cdot P(F) = (\frac{1}{2})(\frac{1}{3}) = \frac{1}{6}$

Since $P(B)\cdot P(F) = P(B \text{ and } F)$ the events are independent.

b) B and E are independent if $P(B)\cdot P(E) = P(B \text{ and } E)$.

$P(B) = \frac{3}{6} = \frac{1}{2}$

$P(E) = \frac{2}{6} = \frac{1}{3}$

$P(B \text{ and } E) = \frac{2}{6} = \frac{1}{3}$

$P(B)\cdot P(E) = (\frac{1}{2})(\frac{1}{3}) = \frac{1}{6}$ Since $P(B)\cdot P(E) \neq P(B \text{ and } E)$, B and E are not independent.

References

Common Core State Standards Initiative. (2010). *Common core state standards for mathematics.* Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.

Illustrative Mathematics

Polya, G. (2014). How to solve it: *A new aspect of mathematical method.* Princeton, NJ: Princeton University Press.

Name_______________________________

Student Page

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