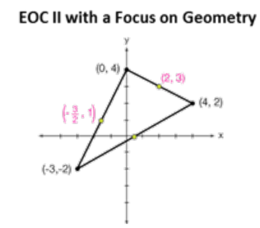
EOC Math II Remediation Lessons



Inscribed Circle

Enduring Understanding

(Do not tell students; they must discover it for themselves.)

Establish and utilize properties of incenters and circumcenters.

Standards

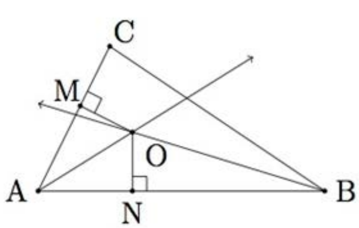
This task might address the following standards (standards might vary based on discussion)
HSG-CO.D.12 **Make geometric constructions** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.)
HSG-CO.C.10 **Prove geometric theorems** Prove theorems about triangles. Theorems include: opposite sides are congruent, opposite angles
HSG-CO.B.8 Understand **congruence in terms of rigid motion** Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
HSG-C.A.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

Launch

Introduce the Task

The goal of this task is to show how to draw a circle which is tangent to all three sides of a given triangle: that is, the circle touches each side of the triangle in a single point.

Suppose *ABC* is a triangle as pictured below with ray $\overrightarrow{AO}$ the bisector of angle *A* and ray $\overrightarrow{BO}$ the bisector of angle *B*:



Nevada Department of Education

Also pictured is the point M on $\overline{AC}$ so that $\overline{OM}$ meets $\overline{AC}$ in a right angle and similarly point N on $\overline{AB}$ is chosen so that $\overline{ON}$ meets $\overline{AB}$ in a right angle.

a) Show that ΔAOM is congruent to ΔAON.

b) Show that $\overline{OM}$ is congruent to $\overline{ON}$.

c) Arguing as in parts (a) and (b) show that if P is the point on $\overline{BC}$ so that $\overline{OP}$ meets $\overline{BC}$ in a right angle then $\overline{OP}$ is also congruent to $\overline{OM}$.

d) Show that the circle with center O and radius $\overline{OM}$ is inscribed inside triangle ABC.

Understand the Problem

- Are there any word(s) you don't understand?
- What is the question or task asking you to answer?
- Is there enough information to find a solution?
- Restate the problem in your own words.
- What additional information do you need to find?

Develop a Plan

- There are many reasonable ways to solve a problem. With practice, students will build the necessary skills to choose an efficient strategy for the given problem.
- Ensure that students have a place to start and that the task/problem has the ability to be scaffolded.
- Caution should be exercised to not force your plan/reasoning on students.

Investigate

Productive Struggle

- Let students engage in productive struggle.
- Monitor as students work.
- Offer positive constructive feedback.
- Ask questions such as…
  - Why did you choose that number?
  - What assumptions did you make?
  - Explain what you are doing here.
  - What does that solution mean?

Questions for Individuals as they Work

If students have a hard time getting started…, then:

Have you marked your diagram?

Have you placed relevant information on the triangle from the paragraph?

If students are not able to formulate the proof of congruence in part a…, then:

In what ways can you prove that triangles are congruent? (SSS, SAS, AAS, ASA, HL)

If students are unable to show congruence between the two segments…, then:

What was determined in part a?

Are the two indicated segments corresponding parts of the congruent triangles?

What do we know about corresponding parts of congruent polygons?

If students are unable to construct $\overline{CO}$ the bisector of……, then:

If you construct $\overline{CO}$, what type of segment have you constructed?

What do you notice about ΔCMO and ΔCPO as compared to the triangles in part a (ΔAMO and ΔANO?)

If students are unable to identify the inscribed circle with center *O* in part d…then:

If $\overline{OM}$ a radius, then what other segments form a radius?

Sample Solutions

Possible Solutions:

a)

Statement	Reason
$\overrightarrow{AO}$ *is a bisector of* $\angle A$	Given
$\angle MAO \cong \angle NAO$	Definition of bisector
$\overline{OM}$ *is perpendicular to* $\overline{AC}$	Given
$\overline{ON}$ *is perpendicular to* $\overline{AB}$	Given
$m\angle AMO = 90°$	Definition of perpendicular
$m\angle ONA = 90°$	Definition of perpendicular
$m\angle AMO = m\angle ONA$	Substitution
$\angle AMO \cong \angle ONA$	Definition of congruence
$\overline{AO} \cong \overline{AO}$	Reflexive property of congruence
$\Delta AOM \cong \Delta AON$	By AAS congruency postulate

b)

Statement	Reason
$\overline{OM} \cong \overline{ON}$	CPCTC

c)

Statement	Reason
Draw $\overrightarrow{CO}$ *as a bisector of* $\angle C$	By construction
$\angle MCO \cong \angle PCO$	Definition of angle bisector
$\overline{OM}$ *is perpendicular to* $\overline{CA}$	Given
$\overline{OP}$ *is perpendicular to* $\overline{CB}$	Given
$m\angle CMO = 90°$	Definition of perpendicular
$m\angle CPO = 90°$	Definition of perpendicular
$m\angle CMO = m\angle CPO$	Substitution
$\angle CMO \cong \angle CPO$	Definition of congruence
$\overline{CO} \cong \overline{CO}$	Reflexive property of congruence
$\Delta COM \cong \Delta COP$	By AAS congruency postulate
$\overline{OM} \cong \overline{OP}$	CPCTC

d) A circle centered at O with radius $\overline{OM}$, and $\overline{OM} \cong \overline{ON} \cong \overline{OP}$ would be inscribed within the triangle whose sides contain points M, N, and P.

Debrief

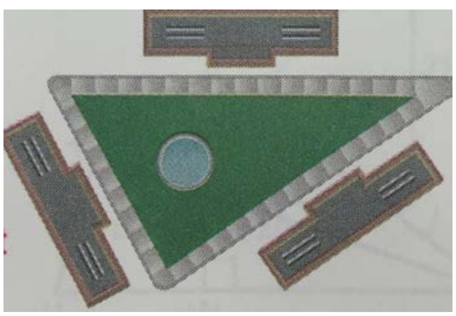
Whole/Large Group Discussion

- Debriefing formats may differ (e.g., whole-class discussion, small-group discussion). It will be beneficial for students to view student work as a gallery walk or similar activity.
- Have students/teacher facilitate the sequence of multiple representations in an order that moves from less to more mathematical sophistication.
- Allow students to question each other and explain their choices, using mathematical reasoning. If students struggle, use questioning strategies.
- Encourage students to notice similarities, differences, and generalizations across strategies.
- Provide constructive feedback and ask clarifying questions for deeper understanding of the process.

Synthesize and Apply

Monitor student work and facilitate discussions by asking questions. When students have independently arrived at the Enduring Understanding, engage them in solving these extension problems. Assess if you have facilitated the discussion in a way that students have arrived at the Enduring Understanding (do not tell them, they will benefit from discovering it for themselves).

Extension Problem #1



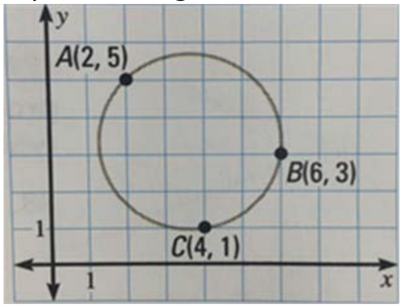
You are designing a circular swimming pool for a triangular lawn surrounded by apartments. You want the center of the pool equidistant from the three sidewalks. Describe how you can locate the center of the pool.

Possible Solution:

Construct angle bisectors for each angle. The point where they intersect is the incenter of the triangle and it is equidistant from three sidewalks

Extension Problem #2

Mycelium fungus grows underground in all directions from a central point. Under certain conditions, mushrooms sprout up in a ring at the edge. The radius of the mushroom ring is an indication of the mycelium's age.

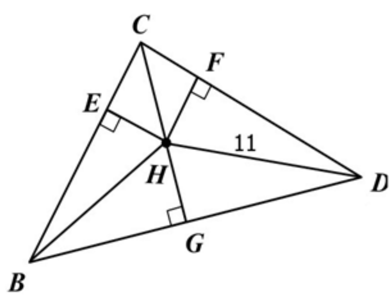


a) Three mushrooms in the ring are marked by points A, B and C. Use triangle ABC to find the center of the ring. And estimate the radius of the ring. Each unit should represent 1 foot.
b) Suppose the radius increases at the rate of 8 inches per year estimate its age.

Possible Solution:

a) Construct angle bisector to the three angles. Use the intersection of the bisectors to locate the center. Draw a radius vertically or horizontally from the center and estimate the length of the radius by measuring with the coordinate grid.
b) Convert feet to inches and use proportions to get about 3.75

Extension Problem #3



If *H* is the incenter of triangle *BCD*,
FD = 14 - 3*x*,
GD = 9*x* - 10,
find the radius of the inscribed circle.

Possible Solution:

Find *x* by setting *FD* = *GD* solve to get *x* = 2.
Find *FD* by substituting 2 for *x* to get *FD* = 8.
Use the Pythagorean Theorem to find the radius, *HF*= $\sqrt{57}$.

References

Common Core State Standards Initiative. (2010). *Common core state standards for mathematics.* Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.

Illustrative Mathematics

Polya, G. (2014). How to solve it: *A new aspect of mathematical method.* Princeton, NJ: Princeton University Press.

Name_______________________________

Student Page

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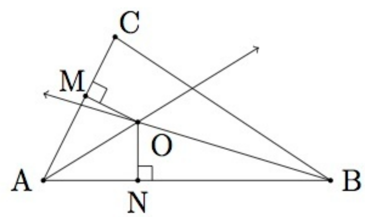

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