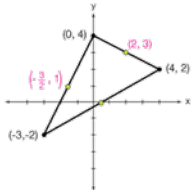


Neglecting the Curvature of the Earth

Enduring Understanding

(Do not tell students; they must discover it for themselves.)

Students will be able to prove theorems about lines and angles and be able to construct a tangent from a point outside the circle to the circle.

Standards

This task might address the following standards (standards might vary based on discussion)
HSG-C.A.2 **Understand and apply theorems about circles.** Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*
HSG-SRT.C.8 **Define trigonometric ratios and solve problems involving right triangles.** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Launch

Introduce the Task

Jerry and Ashley are trying to find out if it is possible to see the lowest point in the USA from the highest point in the USA. It turns out that the highest point in the United States, the peak of Mount Whitney, is only 85 miles from the lowest area, a large flat stretch of land in Death Valley called Badwater Basin. Jerry thinks that the curvature of the earth will block the view of Badwater Basin from the peak of Mount Whitney. Ashley doesn't think that this is an issue and that they can neglect the curvature of the earth in their investigation. As it turns out, it is not possible to see Badwater Basin from Mt. Whitney because the Panamint Range blocks the view. But what if there were no obstacles in the way? The elevation of Mt. Whitney is 14,505 feet and the elevation of Badwater Basin in Death Valley is -282 feet. The radius of the earth is approximately 3959 miles. There are 5280 feet per mile.

More precisely: Standing on top of Mt. Whitney, is it possible to see a point on the surface of the earth 85 miles away, or would the curvature of the earth prevent this?

Understand the Problem

- Are there any word(s) you don't understand?
- What is the question or task asking you to answer?
- Is there enough information to find a solution?
- Restate the problem in your own words.
- What additional information do you need to find?

Develop a Plan

- There are many reasonable ways to solve a problem. With practice, students will build the necessary skills to choose an efficient strategy for the given problem.
- Ensure that students have a place to start and that the task/problem has the ability to be scaffolded.
- Caution should be exercised to not force your plan/reasoning on students.

Investigate

Productive Struggle

- Let students engage in productive struggle.
- Monitor as students work.
- Offer positive constructive feedback.
- Ask questions such as…
    - Why did you choose that number?
    - What assumptions did you make?
    - Explain what you are doing here.
    - What does that solution mean?

Questions for Individuals as they Work

Students do not know the relationship between a radius of a circle and the tangent to that circle… What is the angle formed by the intersection of the radius and the tangent to the circle? (The point of tangency.)

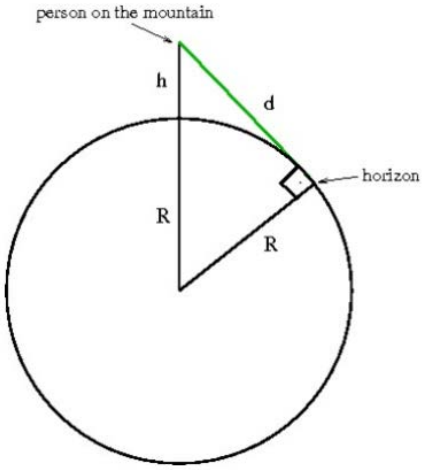
Students do not use the Pythagorean Theorem… How do you find the missing side of a right triangle?

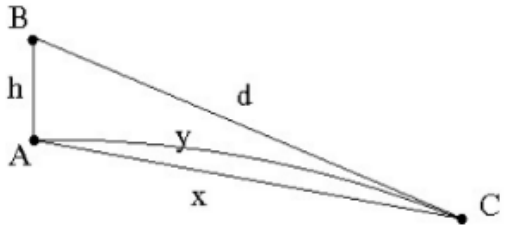
Student does not know where to begin… Have you tried drawing a picture? Label the diagram with what you know.

Students do not draw the correct configuration… How is the tangent to the circle related to the radius at the point of tangency? (The height of the mountain is in addition to the radius of the Earth.)

Sample Solutions

Possible correct answer:





Draw a diagram to determine the point where the line is tangent to the circle. Draw the line segment from the center of the earth to the person on top of the mountain and another line segment from the center of the earth to the point on the horizon. The resulting triangle is a right triangle since the line of sight is tangent to the circle and therefore must be perpendicular to the radius of the circle.
Using the Pythagorean Theorem

$$(R + h)^2 = R^2 + d^2$$
$$(R + h)^2 = R^2 + d^2$$
$$R^2 + 2hR + h^2 = R^2 + d^2$$
$$d^2 = 2hR + h^2$$
$$d = \sqrt{2hR + h^2}$$

The radius of the earth is about 3960 miles, therefore, R = 3960 feet. Mt. Whitney is 14,505 or 2.75 miles high. Thus, $h = 2.75$.
Substituting these values into the equation for we have:

$$d = \sqrt{2 \cdot 2.75 \cdot 3960 + 2.75^2} \quad 147.6$$
$$\approx 148\, miles$$

This means that when we are at the top of Mt. Whitney, we can see locations that are up to 148 miles away.
The distance on the map between Mt. Whitney and Badwater Basin is about 85 miles.
The distance is the length of an arc of a circle. Since the 2.75 mile height of Mt. Whitney is small compared to the distance between the two points on the map and because both of them are really small compared to the radius of the earth, the length of the line of sight, *d*, the length of the arc, *y*, and the length of the chord, *x*, are all very close to each other (within a fraction of a mile). The curvature of the earth would not prevent us from seeing Badwater Basin from the top of Mt. Whitney.

Debrief

Whole/Large Group Discussion

- Debriefing formats may differ (e.g., whole-class discussion, small-group discussion). It will be beneficial for students to view student work as a gallery walk or similar activity.
- Have students/teacher facilitate the sequence of multiple representations in an order that moves from less to more mathematical sophistication.
- Allow students to question each other and explain their choices, using mathematical reasoning. If students struggle, use questioning strategies.
- Encourage students to notice similarities, differences, and generalizations across strategies.
- Provide constructive feedback and ask clarifying questions for deeper understanding of the process.

If you observe this …, you might ask this ….

Students forgetting to take the square root when using the Pythagorean Theorem…. What value do you get from applying the Pythagorean Theorem? What final step must you make to find the value of *c*?
Arithmetic errors… Have you checked your calculations?
Unit conversion errors… Have you checked to make sure your units match?

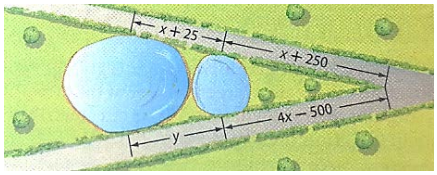
If you see this common error…, it might mean this…

Students miscalculating... They may incorrectly be applying the order of operations.

Synthesize and Apply

Monitor student work and facilitate discussions by asking questions. When students have independently arrived at the Enduring Understanding, engage them in solving these extension problems. Assess if you have facilitated the discussion in a way that students have arrived at the Enduring Understanding (do not tell them, they will benefit from discovering it for themselves).

Extension Problem #1



A landscaper is paving the two walking paths that are tangent to two approximately circular ponds as shown. The lengths are given in feet. Find the values of *x* and *y*.

Possible Solution:

x = 250 feet

y = 275 feet

Extension Problem #2

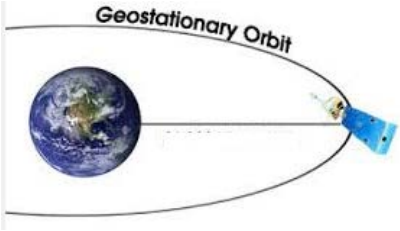
A radio antenna is 450 feet tall. What's the farthest point away on the earth's surface (in miles) that the signal can be broadcast from the antenna?

Assume the radius of the earth as 3959 miles.

Possible Solution:

A signal can be broadcasted about 26 miles.

Extension Problem #3



Direct TV 8, a geostationary satellite is about 35,800 kilometers above Earth. How many arc degrees of the planet are visible to a camera in the satellite?

Assume the radius of the earth is 6,371 kilometers

Possible Solution:

162.6°

References

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Polya, G. (2014). How to solve it: *A new aspect of mathematical method.* Princeton, NJ: Princeton University Press.

Name_____

Student Page

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