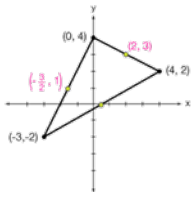


Constructing Special Angles

Enduring Understanding

(Do not tell students; they must discover it for themselves.)

Students will use the trigonometric functions to find the unknown sides and unknown angles of a right triangle. Students will also solve right triangles in applied problems using trigonometric ratios and the Pythagorean Theorem.

Standards

This task might address the following standards (standards might vary based on discussion)

HSG-SRT.C.8 **Define trigonometric ratios and solve problems involving right triangles.** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. *(Modeling Standard)

Launch

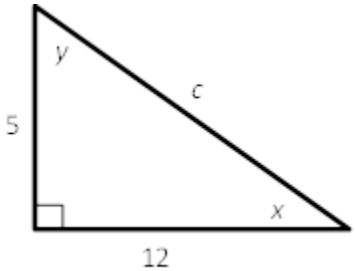
Introduce the Task

The goal of the task is to estimate the measure of angles in triangles with integer side lengths.

a) What are the angle measures in a triangle whose three sides each have length of one unit? Explain.

b) What are the approximate measures of the three angles in a triangle whose side lengths are 3, 4, and 5 units respectively? Explain.

c) What are the approximate values for *x*, *y*, and *c* in the triangle below?



Understand the Problem

- Are there any word(s) you don't understand?
- What is the question or task asking you to answer?
- Is there enough information to find a solution?
- Restate the problem in your own words.
- What additional information do you need to find?

Develop a Plan

- There are many reasonable ways to solve a problem. With practice, students will build the necessary skills to choose an efficient strategy for the given problem.
- Ensure that students have a place to start and that the task/problem has the ability to be scaffolded.
- Caution should be exercised to not force your plan/reasoning on students.

Investigate

Productive Struggle

- Let students engage in productive struggle.
- Monitor as students work.
- Offer positive constructive feedback.
- Ask questions such as…
    - Why did you choose that number?
    - What assumptions did you make?
    - Explain what you are doing here.
    - What does that solution mean?

Questions for Individuals as they Work

Students not knowing how to get started…… Have you drawn your triangles? Have you labeled out all of the information that you know? Have you written out what you know about finding angles for triangles? What prior knowledge do you know about the given data? What does a diagram suggest? What happens when you apply the Pythagorean Theorem? What does it mean when $a^2 + b^2 < c^2$?

Student not being able to label the triangle…… Have you circled the angle you are working with? Which side is that angle making? Which side is that angle not forming? Which side is opposite the right angle? What are those sides called using trigonometric ratios?

Students having trouble setting up the ratio… Can you remember SOH CAH TOA? What does that mean? What does each letter stand for? Can you list and create the ratios for SOH CAH TOA for the referenced angle?

Students not understanding how to solve for an angle … What information comes from using regular trigonometric functions on the calculator? (or on a trig table)? What information comes from using inverse trig functions on the calculator? (or on a trig table)?

Student guesses the angles' measures… How might we use the side ratios to find angles?

Sample Solutions

Possible correct answer:

a) A triangle with three sides of length 1 unit is an equilateral triangle, so all of its angles are congruent and each measure 60°.

b) From ∠*P,* the side adjacent measures 3 units, the side opposite measures 4 units, and the hypotenuse measures 5 units. So, $\sin(P) = \frac{4}{5}$ and $\cos(P) = \frac{3}{5}$. We can determine use them to find the $m(\angle P)$, $m(\angle P) = \sin^{-1}(\frac{4}{5}) \approx 53\circ$. Since $m(\angle P) + m(\angle R) = 90°$, we compute $m(\angle R) \approx 37\circ$ which can also be found using $m(\angle R) = \sin^{-1}(\frac{3}{5})$.

c) To get *c*, use the Pythagorean theorem. $5^2 + 12^2 = c^2$, $25+144 = c^2$, $169 = c^2$, $13 = c$. To solve for *x*, label the sides opposite and adjacent to the angle. $\text{Tan}(x) = \frac{5}{12}$, $\tan^{-1}(\frac{5}{12}) \approx 22.6°$. To get *y*, subtract *x* from 90°, 90°-22.6° = 67.4°, *y* = 67.4°.

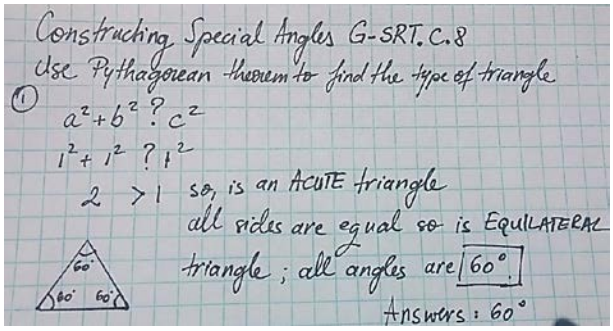
How did you get this answer? Are the angle measures you derived in accordance with the trig ratios you used? To what place did you round your answers and why? Did you check your work?

Debrief

Whole/Large Group Discussion

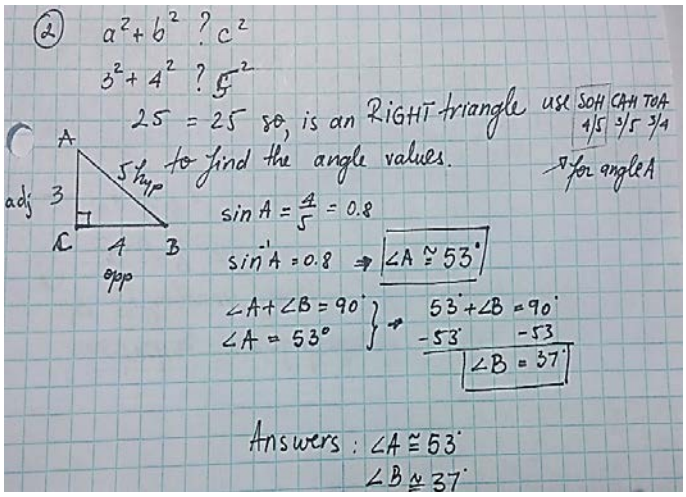
- Debriefing formats may differ (e.g., whole-class discussion, small-group discussion). It will be beneficial for students to view student work as a gallery walk or similar activity.
- Have students/teacher facilitate the sequence of multiple representations in an order that moves from less to more mathematical sophistication.
- Allow students to question each other and explain their choices, using mathematical reasoning. If students struggle, use questioning strategies.
- Encourage students to notice similarities, differences, and generalizations across strategies.
- Provide constructive feedback and ask clarifying questions for deeper understanding of the process.

If you observe this …, you might ask this ….



Sample Student answers

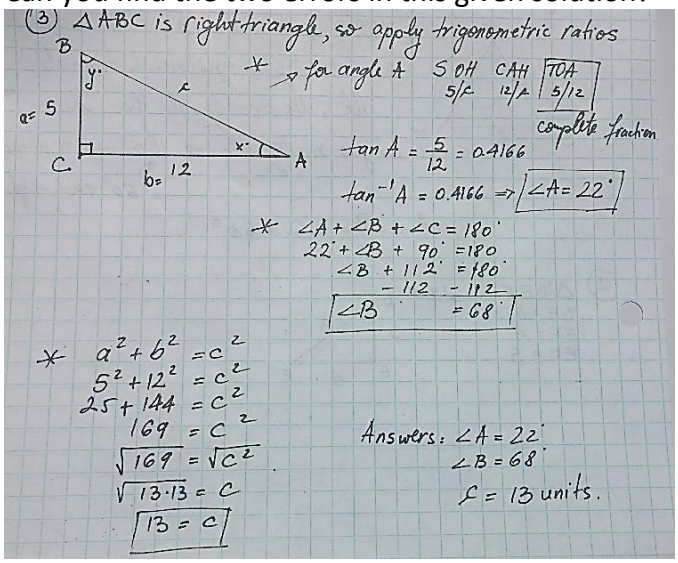
Why should sum of $a^2 + b^2 > c^2$ mean that the triangle is acute?



Sample Student answers

Sin^{-1}(*A)* indicates that students should use the value table for trigonometric functions backward, from the decimal value to discover the angle value.

Can you find the two errors in this given solution?



Students write trig function without an angle… What is required to calculate a sine function?

Student doesn't correctly identify hypotenuse, opposite, and adjacent sides… Have you circled the angle that you are using? Have you labeled the sides correctly? Do your ratios for sine and cosine exceed 1? Have you written SOH CAH TOA?

Students will forget to take the inverse to solve for an angle… What piece of the triangle are you solving for? Does your answer make sense? (can your angle equal 210°?)

If you see this common error…, it might mean this…

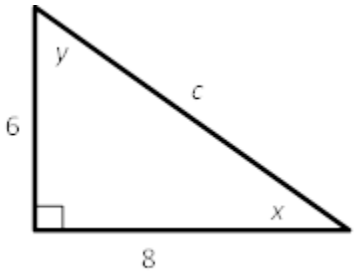
Students write the trig function without an angle.... They may not make a connection between a function and its argument.

Students use the wrong ratio to solve for a side… Student may not have reoriented to the specified angle. They are confusing the hypotenuse with another side. They might have their sides labeled incorrectly or they might always go back to using sine because it is the first one.

Synthesize and Apply

Monitor student work and facilitate discussions by asking questions. When students have independently arrived at the Enduring Understanding, engage them in solving these extension problems. Assess if you have facilitated the discussion in a way that students have arrived at the Enduring Understanding (do not tell them, they will benefit from discovering it for themselves).

Extension Problem #1



The goal of the task is to estimate the measure of angles when given side lengths.

a) What are the approximate measures of the three angles in a triangle whose side lengths are 4, 8, $4\sqrt{3}$? Explain.

b) Solve for the missing information in the triangle above. Find missing side length *c*, and missing angles *x* and *y*.

c) What are the approximate measures of the three angles in a triangle whose side lengths are 12, 16, and 20 units respectively? Show your work.

Possible Solution:

a) $30^\circ, 60^\circ, 90^\circ$

b) c = 10, x = 36.9°, y = 53.1°

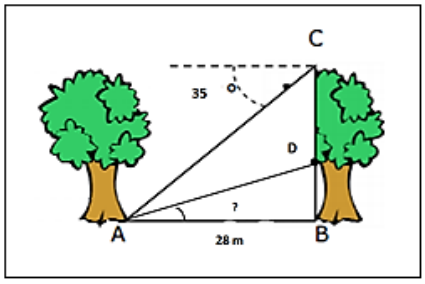
c) $39.9^\circ, 53.1^\circ, 90^\circ$

Extension Problem #2

Two trees face each other separated by a distance of 28 meters. As seen from the top of the second tree, the angle of depression to the first tree's base is 35°.

a) Find the height of the second tree in meters.

b) If the height of the second tree's crown is 8 meters, find the angle of elevation formed by the first tree's base (point *A*) and the bottom of the second tree's crown (point *D*).



c) The second tree will grow two meters in the next five years. What will be the angle of depression formed by the second tree and the first tree's base?

Possible Solution:

a) 90-35 = 55°, tan(55°) = $\frac{28}{BC}$, $(x)\tan(55°) = 28$, $BC = \frac{28}{\tan(55)}$, $BC \approx 19.61$ meters.

b) CD = crown; $BD = BC - CD = 19.61\text{-}8 = 11.61$ meters.

SOH	CAH	TOA
$\frac{5}{?}$	$\frac{28}{?}$	$\frac{5}{11.61}$

use tangent.

$\tan(x^0) = \frac{5}{11.61} \approx 0.4307$, $\tan^{-1}(\frac{5}{11.61}) \approx 23.2998°$, $x \approx 23.3^0$.

c) after five years $BC = 19.61+2 = 21.61$ meters.

The angle of depression from point C to A = the angle of elevation from point A to C. $\tan(y) = \frac{21.61}{28} \approx 0.7718$. If $\tan(y) = 0.7718$ then $y \approx 37.7^0$

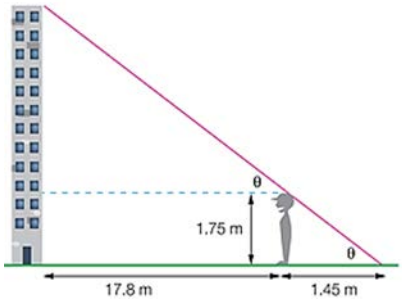
Extension Problem #3

What questions could give these answers?

a) $\tan^{-1}(\frac{1.75}{1.45})$

b) $(17.8+1.45)(\frac{1.75}{1.45})$

c) 39.64°



Possible Solution*:*

a) What is the angle (θ) of elevation? 50.36°

b) How tall is the building? 23.23m

c) What is the measure of the 3rd angle of the triangle?

References

Common Core State Standards Initiative. (2010). *Common core state standards for mathematics.* Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.

Illustrative Mathematics

Polya, G. (2014). How to solve it: *A new aspect of mathematical method.* Princeton, NJ: Princeton University Press.

Name_____

Student Page

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