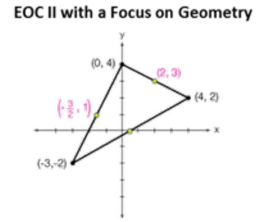
EOC Math II Remediation Lessons



Circles 1

Enduring Understanding

(Do not tell students; they must discover it for themselves.)

Students must know the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

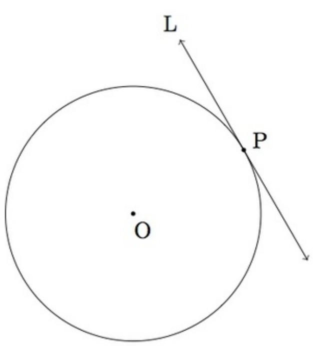
Standards

HSG-C.A.2 **Prove geometric theorems** Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
HSG.SRT.C.8 **Define trigonometric ratios and solve problems involving right triangles** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. *(Modeling Standard)

Launch

Introduce the Task

Consider a circle with center O and let P be a point on the circle. Suppose L is a tangent line to the circle at P that is L meets the circle only at P.



Show that $\overline{OP}$ is perpendicular to L.

Nevada Department of Education

Understand the Problem

- Are there any word(s) you don't understand?
- What is the question or task asking you to answer?
- Is there enough information to find a solution?
- Restate the problem in your own words.
- What additional information do you need to find?

Develop a Plan

- There are many reasonable ways to solve a problem. With practice, students will build the necessary skills to choose an efficient strategy for the given problem.
- Ensure that students have a place to start and that the task/problem has the ability to be scaffolded.
- Caution should be exercised to not force your plan/reasoning on students.

Investigate

Productive Struggle

- Let students engage in productive struggle.
- Monitor as students work.
- Offer positive constructive feedback.
- Ask questions such as…
    - Why did you choose that number?
    - What assumptions did you make?
    - Explain what you are doing here.
    - What does that solution mean?

Questions for Individuals as they Work

If students are confused by the words……, then:
What are the definitions of perpendicular and tangent?
What are the implications of those definitions for the distances of points on the line to the center of the circle?
If students try drawing other line segments…, then:
What might be the lengths of those segments?
How do they compare with the length of segment OP?
If students try making angle measurements…, then:
What other angles could you make using segments drawn to the tangent line from the center of the circle?

If students get stuck in middle of the proof…, then:

Ask students to create steps to solve the problem. (Have them get away from the formality of the proof.)

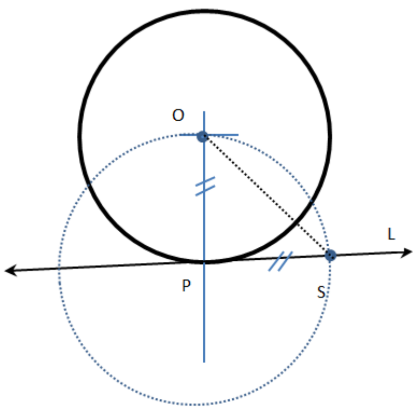
If students do not knowing where to start…, then:

Ask the student for a world application situation (for example: a tire touches a road at exactly one spot; that is tangent and perpendicular. Another example: a soccer ball and a foot. The foot must hit the soccer ball at one point (perpendicularly) to maximize where the ball goes.)

Sample Solutions

Possible Solutions:

Strategy 1



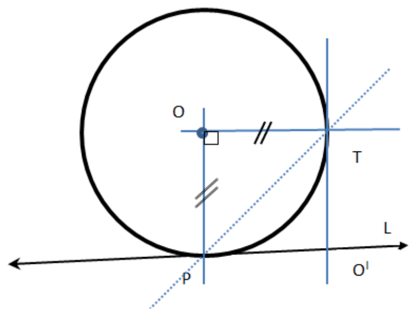
All radii in a circle are equal; draw a circle with center in *P* and radius *PO*;

Point *S* is on line *L* and $\overline{PS}$ is a radius. Triangle *PSO* with $\overline{OP} = \overline{PS}$, is an isosceles triangle. Assume that the triangle is a right triangle with $m\angle P = 90°$. If the assumption is correct then the tangent ratio is true. $\text{Tan}\angle O = \frac{\overline{OP}}{\overline{PS}} = \frac{radius}{radius} = 1$, $\tan^{-1} = 1$, $\angle O = 45°$.

$\text{Tan}\angle S = \frac{\overline{PS}}{\overline{OP}} = \frac{radius}{radius} = 1$, $\tan^{-1} = 1$, $\angle S = 45°$.

In ΔPSO, $\angle O = 45°$, $\angle S = 45°$, and isosceles, so $\angle P = 90°$; therefore the assumption is correct, $\overline{OP}$ is perpendicular to $\overline{PS}$, or line *L*.

Strategy 2



All radii in a circle are equal; draw the radius $\overline{OT}$ such that $\overline{OT}$ and $\overline{OP}$ form a 90^0 angle. Δ*OPT* is isosceles right since $\overline{OP}$ = $\overline{OT}$ (radii) and *m∠O* = 90^o. Draw $\overline{PT}$, the hypotenuse of triangle *OPT.* Reflect point *O* over $\overline{PT}$ and label it *O'*. Δ*O'PT* is similar to Δ*OPT*, therefore also isosceles right. ∠*OPT*=∠*TPO'* = 45°, *m∠OPT* + *m∠TPO'* = 90^O. $\overline{OP}$ is perpendicular, forming 90^0, on line *L*.

Debrief

Whole/Large Group Discussion

- Debriefing formats may differ (e.g., whole-class discussion, small-group discussion). It will be beneficial for students to view student work as a gallery walk or similar activity.
- Have students/teacher facilitate the sequence of multiple representations in an order that moves from less to more mathematical sophistication.
- Allow students to question each other and explain their choices, using mathematical reasoning. If students struggle, use questioning strategies.
- Encourage students to notice similarities, differences, and generalizations across strategies.
- Provide constructive feedback and ask clarifying questions for deeper understanding of the process.

If you observe this …, you might ask this ….

Students not understanding what it means to prove something… What are the characteristics of a proof? When do you know a proof has been completed? Do you have to directly prove something?

If you see this common error…, it might mean this…

Students confusing an assertion as a proof... Students not clear as to how to make connections in a proof.

Students confusing various components of a proof, e.g., statements with reasons… Students not familiar with proof formats.

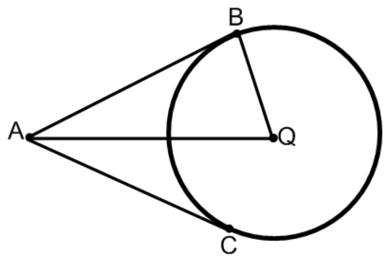
Students use Pythagorean Theorem and they confuse the tangent with the hypotenuse… The tangent is perpendicular to the radius, so the tangent is a leg of the right triangle.

Students don't know the difference between secant, chord and tangent… Secant - a line that passes through the circle; intersects in two points (e.g. an arrow through an apple).Chord - a segment inside the circle whose endpoints are on the circles circumference; it does not continue past the circle's edge (e.g. vocal cord). Tangent - a line that intersect the circle at just one point.

Synthesize and Apply

Monitor student work and facilitate discussions by asking questions. When students have independently arrived at the Enduring Understanding, engage them in solving these extension problems. Assess if you have facilitated the discussion in a way that students have arrived at the Enduring Understanding (do not tell them, they will benefit from discovering it for themselves).

Extension Problem #1

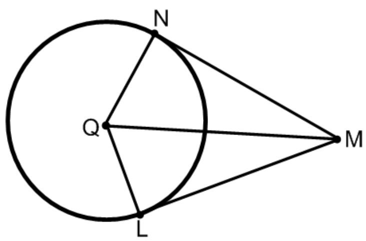


In the diagram above, $\overline{AB}$ and $\overline{AC}$ are tangent to Circle *Q*. Given *BQ* = 4 and *AQ* = 9, calculate *AC*.

Possible Solution:

- $(AB)^2 + (BQ)^2 = (AQ)^2$
- $(AB)^2 + (4)^2 = (9)^2$
- $(AB)^2 + 16 = 81$
- $(AB)^2 = 81 - 16$
- $\sqrt{(AB)^2} = \sqrt{65}$
- $AB = \sqrt{65}$

Extension Problem #2

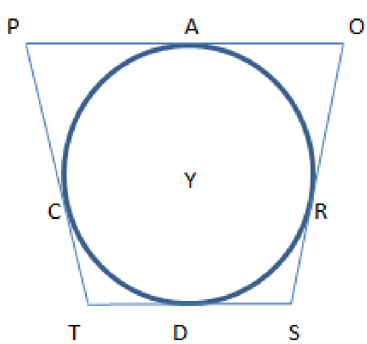


Given: $\overline{LM}$ and $\overline{NM}$ are tangent to Circle Q at points L and N.
Prove: $\overline{LM} \cong \overline{NM}$

Statement	Reason
1. $\overline{LM}$ is tangent to $\odot Q$ at L; $\overline{NM}$ is tangent to $\odot Q$ at N.	1. Given
2. $\overline{QN} \cong \overline{QL}$	2. All radii of a circle are $\cong$
3. $m\angle QNM = m\angle QLM = 90°$	3. The radius drawn to a point of tangency is $\perp$ to the tangent line at that point.
4. $\overline{QM} \cong \overline{QM}$	4. Reflexive Prop.
5. $\triangle QNM \cong \triangle QLM$	5. HL Triangle $\cong$
6. $\overline{LM} \cong \overline{NM}$	6. CPCTC

Extension Problem #3

Quadrilateral *POST* is circumscribed about circle *Y*. *OR* = 13 inches and *ST* = 12 inches. Find the perimeter of *POST*. Note: Point *Y* is the center of the circle and points *A*, *C*, *D*, and *R* are tangent to the circle.



Possible Solution:

OR = *OA* and *OA* = *PA* and *PC* = *PA*. *D* is the midpoint of *ST*. Therefore, *PC*, *PA*, *OA*, and *OR* = 13 and *TC*, *TD*, *SD*, and *SR* = 6. The total is 4 (13) + 4(6) = 76 inches.

References

Common Core State Standards Initiative. (2010). *Common core state standards for mathematics.* Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.

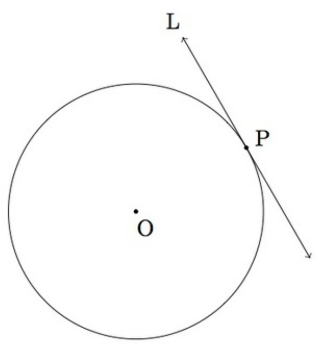
Illustrative Mathematics

Polya, G. (2014). How to solve it: *A new aspect of mathematical method.* Princeton, NJ: Princeton University Press.

Name________________________________

Student Page

Consider a circle with center *O* and let *P* be a point on the circle. Suppose *L* is a tangent line to the circle at *P*, that is *L* meets the circle only at *P*.



Show that $\overline{OP}$ is perpendicular to *L*.