

# Congruence of Parallelograms

## Enduring Understanding

**(Do not tell students; they must discover it for themselves.)**

Students will be able to know and utilize the 5 properties of a parallelogram.

### Standards

**HSG-CO.C.11 Prove geometric theorems** Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

**HSG-SRT.A.2 Understand similarity in terms of similarity transformations** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

**HSG-SRT.A.3 Understand similarity in terms of similarity transformations** Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

**HSG-CO.B.8 Understand congruence in terms of rigid motions** Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

**HSG-SRT.B.5 Prove theorems involving similarity** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

## Launch

### Introduce the Task

Rhianna has learned the SSS and SAS congruence tests for triangles and she wonders if these tests might work for parallelograms.

- Suppose  $ABCD$  and  $EFGH$  are two parallelograms all of whose corresponding sides are congruent, that is  $AB = EF$ ,  $BC = FG$ ,  $CD = GH$ , and  $DA = HE$ . Is it always true that  $ABCD$  is congruent to  $EFGH$ ?
- Suppose  $ABCD$  and  $EFGH$  are two parallelograms with a pair of congruent corresponding sides,  $AB = EF$  and  $BC = FG$ . Suppose also that the included angles are congruent,  $m\angle ABC = m\angle EFG$ . Are  $ABCD$  and  $EFGH$  congruent?

## Debrief

### Understand the Problem

- Are there any word(s) you don't understand?
- What is the question or task asking you to answer?
- Is there enough information to find a solution?
- Restate the problem in your own words.
- What additional information do you need to find?

### Develop a Plan

- There are many reasonable ways to solve a problem. With practice, students will build the necessary skills to choose an efficient strategy for the given problem.
- Ensure that students have a place to start and that the task/problem has the ability to be scaffolded.
- Caution should be exercised to not force your plan/reasoning on students.

## Investigate

### Productive Struggle

- Let students engage in productive struggle.
- Monitor as students work.
- Offer positive constructive feedback.
- Ask questions such as...
  - Why did you choose that number?
  - What assumptions did you make?
  - Explain what you are doing here.
  - What does that solution mean?

## Questions for Individuals as they Work

**Students did not match the correct corresponding sides...**

Does  $AB = EF$ ?

What is required for two parallelograms to be congruent?

**Student incorrectly drawing a parallelogram; Opposite sides are not parallel or congruent....**

Did you consider all properties of a parallelogram?

**Students may make incorrect inquiries concerning drawing parallelograms...**

Do all parallelograms have to be congruent?

**Students making incorrect assumptions concerning congruent parallelograms...**

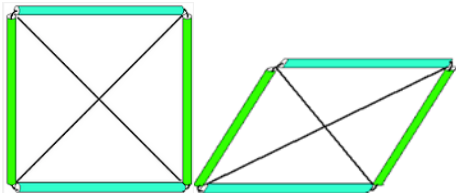
How many corresponding sides/angles are necessary to prove parallelograms are congruent?

**Student is drawing triangles instead of parallelograms...**

Why did you draw a triangle?

How many sides does a parallelogram have?

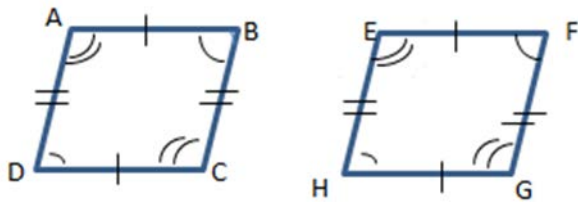
**Student drawings look like this...**



If students have difficulty comparing the two parallelograms use string and straw to help students develop the correct concept. See illustration.

## Sample Solutions

Possible correct response:



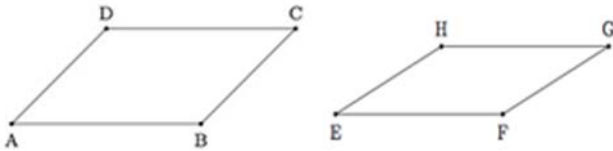
- Opposite sides of parallelograms are congruent.
- Opposite angles of parallelograms are congruent.
- Adjacent angles of parallelograms are supplementary.
- Diagonals bisect each other.
- One pair of opposite sides is both parallel and congruent.

## Debrief

### Whole/Large Group Discussion

- Debriefing formats may differ (e.g., whole-class discussion, small-group discussion). It will be beneficial for students to view student work as a gallery walk or similar activity.
- Have students/teacher facilitate the sequence of multiple representations in an order that moves from less to more mathematical sophistication.
- Allow students to question each other and explain their choices, using mathematical reasoning. If students struggle, use questioning strategies.
- Encourage students to notice similarities, differences, and generalizations across strategies.
- Provide constructive feedback and ask clarifying questions for deeper understanding of the process.

### If you observe this ..., you might ask this ....



SSSS does not exist as a method to prove that parallelograms are congruent.

$$AC \neq EG$$

$$\square ABCD \not\cong \square EFGH$$

### If you see this common error..., it might mean this...

Students trying to show adjacent sides are perpendicular...They are only looking at special quadrilaterals.

Students calculate the slope incorrectly...

They used  $\frac{\text{Run}}{\text{Rise}}$ , not  $\frac{\text{Rise}}{\text{Run}}$ .

Student miscalculates the distance of the sides... Students forgot to take the square root.

Students added the vertices instead of subtracting.

Students assume diagonals are congruent... Diagonals bisect but are not congruent.

Miscalculations... Arithmetic errors

## Synthesize and Apply

Monitor student work and facilitate discussions by asking questions. When students have independently arrived at the Enduring Understanding, engage them in solving these extension problems. Assess if you have facilitated the discussion in a way that students have arrived at the Enduring Understanding (do not tell them, they will benefit from discovering it for themselves).

### Extension Problem #1

Given: Parallelograms  $ABCD$  and  $EFGH$   
 $\angle ABC \cong \angle EFG$ ;  $\overline{BC} \cong \overline{FG}$ ;  $\angle BCD \cong \angle FGH$

Could you use ASA to prove two parallelograms are congruent?

### Possible Solutions:

No, because  $AB \neq EF$

### Extension Problem #2

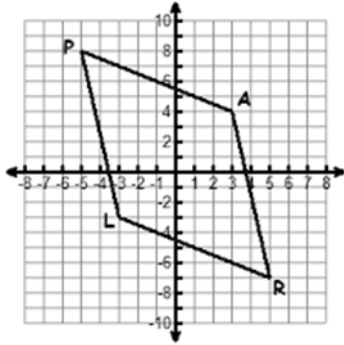


While building a hand rail, a carpenter uses a plumb bob to confirm that the vertical supports are perpendicular to the top step and the ground respectively. How can the carpenter prove that the two hand rails are parallel using the fewest measurements? Assume that the top step and the ground are both level.

### Possible Solutions:

Since two vertical rails are both perpendicular to the ground, he knows that they are parallel to each other. If he measures the distance between the two rails at the top of the steps and at the bottom of the steps, and they are equal, then one pair of sides of the quadrilateral formed by the handrails is both parallel and congruent, so the quadrilateral is a parallelogram. Since the quadrilateral is a parallelogram, the two hand rails are parallel by definition.

### Extension Problem #3



Given quadrilateral  $APLR$  above:  
show two different methods of proving that  $APLR$  is a parallelogram.

#### Possible Solutions:

To determine whether  $APRL$  is a parallelogram, here are two possible methods. There are more possible methods.

##### Method 1

$$d_{AP} = \sqrt{80}$$

$$d_{LR} = \sqrt{80}$$

$$d_{PL} = \sqrt{125}$$

$$d_{AR} = \sqrt{125}$$

##### Method 2

$$m_{AP} =$$

$$\frac{4}{-8} = -\frac{1}{2}$$

$$m_{LR} =$$

$$\frac{-4}{8} = -\frac{1}{2}$$

$$d_{AP} = \sqrt{80}$$

$$d_{LR} = \sqrt{80}$$

## References

Common Core State Standards Initiative. (2010). *Common core state standards for mathematics*. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.

### [Illustrative Mathematics](#)

Polya, G. (2014). *How to solve it: A new aspect of mathematical method*. Princeton, NJ: Princeton University Press.

