

**Proposal**

**Statewide Coordinating Council  
of the  
Regional Professional Development Programs**

**Evaluating & Supervising Math Workshops  
for  
Administrators**

**Mathematical Systems, Inc.**

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## *The Need*

The national research concerning teacher evaluations is compelling. That is, over 60% of school administrators do not confront poor instruction and over 75% of teachers surveyed indicate that their evaluations have little impact on their instruction.

It's no secret that Nevada's proficiency rates in math are dismal. The state has experienced a documented math teacher shortage since 1985 – over 35 years. That shortage has been exacerbated since the pandemic. Understanding that student success in science, engineering and technology is dependent upon student success in math makes this not only a concern for Nevada, but for the nation.

The teacher shortage has resulted in far too many students being taught not only by underqualified math teachers, but also by people just covering classes. That alone makes math a lot more difficult to learn for our students.

While people in the community believe math teachers are being evaluated and supervised by administrators who provide recommendations that will improve instruction, we know that is not true. The great majority of administrators evaluating and supervising math know a lot less math than the under-qualified teachers they are observing.

Reading teacher evaluations strongly suggests school administrators generally utilize “instructional” strategies in their role as supervisors. While instructional strategies are important, they do not actually address the math content or how to present that content to students in understandable terms. In fact, school administrators do not possess the knowledge or understanding of math content or strategies that will result in providing suggestions, recommendations, or directions to teachers to make math easier for students to learn while actually increasing student achievement.

To increase student achievement in math, especially since the state is experiencing a shortage of well qualified teachers, professional development for administrators is necessary for them to be able to assist their teachers resulting in increased student achievement in math.

## **Bill Hanlon**

Former Director of the Southern Nevada Regional Professional Development Program, was a noted speaker, author, educator, consultant and coach for schools, and was a national presenter for organizations such as AASA, ASCD, ALAS, NMSA, NASSP, NSBA, and NCTM.

Bill was also the coordinator of Clark County School District's Math/Science Institute and was also responsible for K-12 math audits. He served on the Nevada State Board of Education, Regional Director of the National Association of State Boards of Education (NASBE) and as a member of the National Council for Accreditation of Teacher Education (NCATE) States Partnership Board. He also hosted a television series, "Algebra, you can do it!" on PBS Las Vegas.

His experiences include teaching for twenty years at the college, high school and junior high school levels as well as working with elementary teachers and administrators. As a classroom teacher, Bill served as department chair at Orr Junior High and Eldorado High School. At Eldorado, he also served as the advisor to the National Honor Society, Key Club and Student Council. While teaching a wide variety of courses, Bill concentrated on working with struggling students, and students living in poverty.

Bill is president of Mathematical Systems, Inc. Through his company he has successfully provided professional development and guidance to struggling schools and districts throughout the country. His website, [hanlonmath.com](http://hanlonmath.com), provides resources for teachers that include videos, chapters, booklets, and worksheets that are specifically designed and created for students who traditionally struggle in mathematics.

Bill's books, "Building Success on Success" and "Teaching Struggling Students in Math" can be purchased through Rowman & Littlefield.

## *Proposal*

Mathematical Systems, Inc., Bill Hanlon, will offer thirty Evaluating & Supervising Math Workshops for administrators that address math content and strategies to teach that content. The target audience of the Evaluating & Supervising Math zoom math workshops is at least one school administrator per school in Nevada, approximately 900 participants. These workshops concentrate on specific major topics in math addressing the “missing math content” strategies on teacher observations and formal evaluations.

The Evaluation & Supervision of Math proposal is research based that effects administrators’ knowledge, understanding, skills and leadership behaviors at the school and classroom levels that will improve instruction and result in increased student achievement in mathematics. It should be noted that success in science, engineering and technology depend upon a successful math program.

In addition, the Evaluation & Supervision of math workshops complement and support workshops offered for teachers. These workshops directly impact the ability of administrators to effectively communicate with teachers that impact student competency in mathematics.

It should be noted the greatest differences between the math taught in elementary school to the math taught in secondary school are vocabulary, notation, and pattern development. Examples of those linkages can be found in the appendix. Those linkages allow teachers to review and reinforce knowledge and/or address student deficiencies as they teach their assigned curriculum. It also allows teachers the opportunity to introduce new topics in a familiar language which makes students more comfortable in the new learning.

As many know, nothing ruins a good math lesson like a bad example. Administrators will be provided simple straight-forward examples that work, without variation, that don’t bog students down in needless arithmetic, so students are focused on the concept or skill being taught. In addition to those initial examples, examples for repeated scaffolding will be provided so students reach grade level expectations.

The specific “*math content*” strategies are introduced through actual math examples that are used in the classroom and will provide school administrators greater insight on how to truly improve math instruction, student understanding and comfort levels when new concepts and skills are being introduced. The

handouts are made available on the Hanlonmath website before each workshop. The emphasis will be placed on linkages, conceptual development, examples of simple straight-forward examples on initial instruction, and repeated scaffolding examples to reach grade level expectations.

Other *math content* strategies, such as the importance of definitions, looking for patterns, comparing and contrasting, thinking out loud, using models, using correct notation and the importance of decision-making when doing math, will also be addressed. In each workshop, administrators will receive a booklet of those recommendations and a practice test specifically designed to set students up for success.

There are three specific intended outcomes, 1. increasing school administrators ability to provide suggestions, recommendations or directions to teachers based on the actual content being presented, 2. changing how teacher observations are scheduled. Rather than picking random dates for observations, administrators would choose dates based on their ability to provide input on the actual math content being presented. And importantly, 3. improved instruction will result in student success which positively impacts students' self-confidence and emotional development.

At the completion of each workshop each participant will receive a summary of the workshop and a booklet containing those suggestions and recommendations on how to introduce "new" topics by linking them to prior knowledge or outside experiences, examples to be used with initial instruction, repeated scaffolding examples to be used to reach grade level expectations, and an example of the unit or chapter test designed to set students up for success and be better prepared for high stakes state or national tests.

Additionally, these workshops will strongly suggest that rather than identifying a date to observe instruction based on an open calendar, school administrators should schedule observations based on the introduction of new content they are familiar with so they are better able to address math content strategies to improve instruction.

To determine the effectiveness of the proposal, participants will evaluate each workshop attended, surveys will be sent to administrators in March to determine if they actually changed dates of observations based on their knowledge of content they were observing, and teachers will be surveyed to determine if the evaluation

or post observation conference with their administrator discussed the math content standards.

The following workshops will be offered with others based upon request.

<b>Basic Math Facts</b>	<b>Exponentials</b>	<b>Quadratic Equations</b>
<b>Properties of Real</b>	<b>Area-Perimeter-Volume</b>	<b>Eqns. of Lines</b>
<b>Numbers</b>	<b>Angle Measurement</b>	<b>Graphing Linear</b>
<b>Fractions</b>	<b>Intro Right Triangles</b>	<b>Equations</b>
<b>Decimals</b>	<b>Intro Trig</b>	<b>Graphing Quadratic</b>
<b>Integers</b>	<b>Linear Equations &amp;</b>	<b>Equations.</b>
<b>Ratio &amp; Proportion</b>	<b>Inequalities</b>	<b>Exponential Equations</b>
<b>Percents</b>	<b>Alg. Word Problems-1V</b>	<b>Logarithmic Equations</b>
<b>Intro Probability</b>	<b>Polynomial Operations</b>	<b>Building Success on</b>
<b>Intro Statistics</b>	<b>Factoring Polynomials</b>	<b>Success</b>
	<b>Systems of Equations</b>	

At the conclusion of each workshop, school administrators will have a much greater ability to improve math instruction by actually discussing how the content is being delivered by the teacher being observed and evaluated. That is, they will be able to identify linkages, correct definitions and notations, provide simple, straight-forward examples to be used with initial instruction to enhance student understanding and comfort levels as well as provide repeated scaffolding examples for students to reach grade level expectations.

**Booklet**

**Evaluating & Supervising Math Booklet**



***Recommendation Booklet***  
**Solving Linear Equations & Inequalities**

**Sample**  
**Suggestions & Recommendations**  
*for initial instruction*  
**Designed for Formal & Informal Evaluations**  
**By School Administrators**  
**that**  
**Address Math Content Strategies**



## *Addressing the needs of struggling students in math*

**Nevada has experienced a documented math teacher shortage since 1985. We are hiring less and less qualified teachers to fill vacant positions. Those less qualified are being supervised and evaluated by school administrators who know less math than the teachers they are supervising and rely on “instructional” strategies in their evaluations. To actually improve instruction that directly address math achievement, these booklets and associated workshops focus on “math content” strategies that directly impact what teachers teach, how they teach it, and assessments that support the understanding of mathematics.**

**The contents of this booklet contain specific math content strategies that could be used in the supervision and evaluation of math teachers. Those math content strategies include linkages to previously learned material or outside experiences that provide opportunities to review, reinforce or address student deficiencies as they introduce new material. Those linkages also allow teachers to introduce new concepts & skills in more familiar language which makes students more comfortable learning math.**

**Also provided in this booklet are simple, straight-forward examples that could be suggested to be used by teachers when first introducing a topic. These examples do not distract or bog students down in needless arithmetic – they keep the focus on the new learning. These problems are chosen specifically to build improve student confidence by building success on success. Those examples are followed by practice problems for students using numbers that don’t distract students.**

**There are examples of practice problems that administrators could suggest to be used when scaffolding – again using simple straight, forward examples.**

**In addition, there are also strategies, procedures, and/or formulas that might be recommended to teachers so students have a written guide, with examples, as they process what they are learning and put that knowledge into words – language acquisition.**

**And finally, a sample test is included that is specifically designed to set students up for success.**

## Linear Equations & Inequalities – Evaluation & Supervision

### LINEAR EQUATIONS – 1 VARIABLE

#### RECOMMENDATION

A suggestion would be to review the Order of Operations, then using examples provided below that require division before multiplication to emphasize multiplication and division have the same rank and must be done in order from left to right. Explain, not having everyone doing the operations in the same order would result in different answers. The importance of the having an agreement – the Order of Operations – so everyone does the problem the same way and arrives at the same answer.

30

$2 + 4 \times 5 = <$             2 reasonable answers – only one can be correct

22

**Order of Operations – just an agreement like driving on the right side of the road so we all get the same answer.**

    Parentheses  
    Exponentials  
    Multiply/Divide } From Left  
    Add/Subtract    } to Right

#### RECOMMENDATION

**Provide a few examples in which division comes first to address common misconceptions.**

#### EXAMPLES

Simplify the following: (emphasizing division can come first!)

$$2 + 20 \div 2 \times 5 + 1$$

$$4 + 24 \div 6 \times 2 - 3$$

$$20 - 12 \div 2 \times 3 + 2$$

#### RECOMMENDATION

**Ask students the following question; if it costs \$10.00 to enter the amusement park and \$5.00 per ride, how many rides can go on if you have \$90.00?**

**After students provide the answer using mental math, show them how they actually used algebra to solve the problem.  $\$10 + \$5r = \$90$**

## RECOMMENDATION

Consider quickly introducing simple equations used in elementary school such as using  $4 + \Delta = 6$  where they had to find the value that goes into the  $\Delta$  by guessing and substituting numbers to find a number that worked. Then scaffold to a problem such as  $2 \times \Delta + 4 = 14$ .

## EXAMPLES

Fill in the missing number – Guess & Check

$$5 \times \Delta + 3 = 13$$

$$10 \times \Delta + 2 = 52$$

Using the elementary example,  $2 \times \Delta + 4 = 14$ , show them how that looks in algebra.

$2x + 4 = 14$ . The only change being vocabulary & notation.

## RECOMMENDATION

Go over the Gift Wrapping Analogy and show how that is used to develop a systematic approach to solving equations in algebra.

Based on the Gift Wrapping Analogy and the Order of Operations, provide students with a systematic strategy for solving all linear equations and have them write it in their notes.

More systematic approach for equations already in  $ax + b = c$  format

## Order of Operations

↑  
Parentheses  
Exponentials  
Multiply/Divide  
Add/Subtract

## RECOMMENDATION

Have the students write the following strategy in their notes

### Strategy for Solving Linear Equations

*Rewrite an equation in  $ax + b = c$  format using the Properties of Real Numbers, then use the Order of Operations in reverse using the inverse operations to isolate the variable.*

*a. Identify what is physically different from  $ax + b = c$*

*b. Get rid of it*

Redo the previous elementary equations using algebraic notation.

#### **EXAMPLES**

$$5x + 3 = 13$$

$$10x + 2 = 52$$

$$3x - 7 = 5$$

$$x/4 - 3 = 2$$

#### **PRACTICE PROBLEMS**

$$4x - 3 = 17$$

$$10x + 4 = 54$$

$$5x + 6 = 36$$

$$x/2 - 3 = 7$$

**Important formulas, procedures or strategies, such as the Order of Operations should be written and left on the board for reference**

When introducing equations, use numbers that don't distract students from the concept or skill being taught or bog them down in arithmetic.

#### **RECOMMENDATION**

After the students are comfortable solving equations using the before mentioned strategy, then have them begin to justify each step.

On high stakes tests, students must justify their answers. So after the students are comfortable with solving simple equations, do a couple of more examples by providing justifications making sure the equations solved are horizontally.

$$\begin{aligned}7x + 3 &= 31 \\7x + 3 - 3 &= 31 - 3 \\7x + 0 &= 28 \\7x &= 28 \\x &= 4\end{aligned}$$

Given  
Subtract Prop Equality  
Add Inverse/Combine terms  
Property of Zero  
Div. Prop Equality

Continually reference the strategy developed when doing problems.

### **SCAFFOLDING - Variables on Both Sides**

#### **RECOMMENDATION**

Explain that problems cannot be made more difficult – only longer. Scaffold up by providing a couple of examples to be worked out for them using the *Get Rid of It* strategy. Examples such as  $5x - 2 = 2x + 19$  and  $7x + 3 = 3x + 23$ . Have students identify how these equations look physically different, by underlining or circling, than the equations solved previously – comparing and contrasting.

Use the strategy for solving linear equations, step by step, continually referring to that strategy as you are doing each step by speaking out loud as you proceed.

#### **EXAMPLES**

Do equations with variables on both sides following strategy written on board;

$$8x - 2 = 3x + 28$$

$$10x + 4 = 7x + 25$$

$$7x - 2 = 3x + 30$$

#### **PRACTICE PROBLEMS**

Solve for x:

$$5x + 3 = 2x + 18$$

$$7x - 2 = 4x + 19$$

$$8x + 6 = 3x + 21$$

#### **RECOMMENDATION**

As you continue to scaffold, continue to reference the strategy for solving linear equations, that is rewrite the equations into  $ax + b = c$  format using the *Get Rid of It* strategy. Taking something you don't recognize and changing that into a pattern you do recognize. Comparing and contrasting the equations. Introduce equations containing parentheses by asking what is physically different, then have the students follow the *Get Rid of It* strategy.

### **SCAFFOLD AGAIN– Distributive Property**

#### **EXAMPLES**

Solve for x

$$4(3x - 2) - 2x = 22$$

$$3(2x + 1) - 4 = 11$$

$$5(2x + 3) - 4 = 21$$

## PRACTICE PROBLEMS

$$5(2x - 3) + 5 = 20$$

$$2(x - 3) - 3 = 3x + 2$$

$$5(2x + 3) - 2(x - 4) = 2x - 1$$

## RECOMMENDATION

After the students are comfortable with solving equations with variables on both sides of the equation and the Distributive Property, have them then do a couple of longer problems.

## EXAMPLES

$$2x + 3(2x + 5) = 39$$

$$2(x - 3) - 3 = 3x + 2$$

$$5(2x + 3) - 4(x - 2) = 3x + 26$$

## PRACTICE PROBLEMS

$$3(2x - 3) + 4x = 5x + 16$$

$$4(3x - 2) - 2x = 22$$

$$5(2x + 3) - 2(x - 4) = 2x - 1$$

## RECOMMENDATION

After the students are comfortable and successfully solving equations, have them redo a few problems with justifications for each step

## EXAMPLE

$6x + 8$	$= 2x + 4$	Given
$6x + 8 - 2x$	$= 2x + 4 - 2x$	Sub. Prop of Equality (SPE)
$4x + 8$	$= 4$	Add. Inv. / CLT
$4x + 8 - 8$	$= 4 - 8$	SPE
$4x + 0$	$= -4$	Add. Inv. / CLT
$4x$	$= -4$	Prop of Zero or Add Identity
$x$	$= -1$	DPE

## PRACTICE PROBLEMS

$$2x + 3(2x + 5) = 39$$

$$2(x - 3) - 3 = 3x + 2$$

$$5(2x + 3) - 4(x - 2) = 3x + 26$$

## INEQUALITIES

### RECOMMENDATION

To clarify solving inequalities when multiplying or dividing by a negative number, give students a few examples using a number line stressing numbers to the right on a number line are greater than numbers to the left.

If  $a > b$ , then  $a + c > b + c$ .

If  $5 > 2$ , then  $5(3) > 2(3)$  —  $\geq$  if  $a > b$ , then  $ac > bc$

However, when multiplying /dividing by a negative number If  $5 > 2$ , then  $5(-3) ? 2(-3)$

if  $c < 0$ , and  $a > b$ , then  $ac < bc$

When we multiply or divide by a negative number, to make the statement true, the order of the inequality must be reversed.

### RECOMMENDATION

Write that finding as a rule. When multiplying or dividing by a negative number, reverse the order of the inequality.

### RECOMMENDATION

Use same numbers in inequalities that were used in equalities so students know the strategy does not change.

### EXAMPLES

**Inequalities – (same procedure as equalities)**

$$5x + 3 < 2x + 18$$

$$7x - 2 \geq 4x + 19$$

$$8x + 6 \leq 3x + 21$$

### RECOMMENDATION

After students recognize that inequalities are essentially solved the same way as equalities, then do two examples where the order of the inequality changes because of either multiplication or division by a negative number

### EXAMPLES

Inequalities with negative number

$$-2x + 3 < 13$$

$$7x - 2 \geq 10x + 19$$

### PRACTICE PROBLEMS

$$-4x + 2 > 30$$

$$8x - 2 \leq 10x + 6$$
$$10 - 4(x + 3) \geq 2x + 4$$

### **RECOMMENDATION**

Have the students write the following procedure in their notes

1. Rewrite the inequality in  $ax + b > c$  format
2. Isolate the variable using inverse operation using the Order of Operations in reverse
3. If you multiply or divide the inequality by a negative number, reverse the order of the inequality.

### **PRACTICE PROBLEMS**

$$-4x + 3 \geq 23$$
$$-2x + 3 < 8x + 33$$
$$4(2x - 3) + 5 > 5(3x + 7)$$

### **COMPOUND (DOUBLE) INEQUALITIES**

#### **RECOMMENDATION**

When reading a double inequality, read the variable expression first, then read the inequalities using “and”. So,  $-2 < x \leq 4$  would be read,  $x$  is less than or equal to 4 **AND** greater than  $-2$ .

Emphasize “and” and “or” statements with double inequalities and how an “and” statement occurs when the graphs overlap.

Stress solving inequalities and double inequalities follows our same strategy, rewrite the double inequality as two separate inequalities

#### **RECOMMENDATION**

Require students to write the following strategy in their notes.

#### **Solving Double Inequalities**

To solve a double inequality, you solve the two inequalities independently, then use the “and” or “or” statement to determine the solution set. In other words, solve the middle to the right of the inequality, then solve from the middle to the left.



**Procedure:**

1. Determine if the compound inequality is connected by “and” or “or”.
2. Solve each inequality
3. If compound inequality is connected by an “and”, then the solution must satisfy each statement. The solution is where the graphs of each overlap.  
  
3a. If the compound inequality is connected by an “or” statement, then the solution can satisfy either or both statements. The solution is the graph of both solutions.

**EXAMPLES**

Solve for x;

$$-11 < 2x + 1 < 7$$

$$-13 \leq 2x + 3 < 5$$

$$x + 1 < 3 \text{ or } 2x - 1 \geq 7$$

**PRACTICE PROBLEMS**

$$12 < 2x \leq 16$$

$$-3 \leq 2x + 1 < 9$$

$$2x - 1 > 9 \text{ or } 3x + 2 < 5$$

## ABSOLUTE VALUE EQUATIONS

Introduce absolute value as students learned from elementary school, then emphasize in algebra, we are looking for all the values of a variable that makes an open sentence true and therefore must have a more precise definition. We define absolute value as a piecewise function.

$$|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

To increase student understanding, make the arguments simple,  $|x|$ , and scaffold to make longer problems. Then use the definition to the two possibilities – when the argument is positive and when the argument is negative.

Use the following procedure to solve equations containing absolute value using the get rid of it

### RECOMMENDATION

#### Solving Equations with Absolute Value

1. Isolate the absolute value
2. Set the positive and negative of the expression inside the absolute value signs equal to the number on the outside creating 2 equations
3. Solve the resulting equations in the  $ax + b = c$  format

### EXAMPLES

$$|x| = 8$$

$$|x - 1| = 12$$

$$|2x + 1| = 13$$

### PRACTICE PROBLEMS

$$|2x + 3| = 13$$

$$|2x - 1| = 9$$

$$|3(x - 2)| = 12$$

$$2|2x - 1| - 4 = 8$$

**Caution – if the definition of absolute value is not stressed and the first step is skipped, then students will experience difficulty when solving inequalities.**

## **ABSOLUTE VALUE INEQUALITIES**

### **RECOMMENDATION**

To determine if an absolute value inequality is an “and” or an “or” statement, ask students to identify what numbers would work for  $|x| > 3$  and  $|x| \leq 2$ . Then summarize by indicating when the absolute value is less than, it’s an “and” statement, when its greater then, it’s an “or” statement.

Do the same absolute value problems from above but use inequalities.

### **EXAMPLES**

$$|x| > 8$$

$$|x - 1| \geq 12$$

$$|2x + 1| < 13$$

### **PRACTICE PROBLEMS**

$$|2x + 3| < 13$$

$$|2x - 1| \geq 9$$

$$|3(x - 2)| < 12$$

$$2|2x - 1| - 4 \leq 8$$

## **TESTING**

### **RECOMMENDATION**

Construct a test specifically designed to increase student performance using our test template with 3-star, 2-star and 1-star questions. The 3-star questions have no computation or manipulation and are reviewed daily during the QCPR and are on the test. The 2-star questions are problems based on the 3-star that are checked for proficiency everyday right after the QCPR. And the 1-star questions are ACT/SAT, conceptual type questions. A copy of the practice test should be constructed and posted on the math department’s website so students and parents have access.

1. \*\*\*Using mathematical notation, define the Distributive Property.
  
2. \*\*\*Using mathematical notation, define Absolute Value.
  
3. \*\*\*Write the following in word form.  $3 < 2x + 1 \leq 10$
  
4. \*\*\*Write the Order of Operations
  
5. \*\*\*Write the strategy for solving linear equations.
  
6. \*\*\*Write the strategy for solving equations containing absolute value.
  
7. \*\*  $4x + 2 = 22$
8. \*\*  $\frac{2x}{3} - 4 = 10$

9. \*\*  $8x + 6 = 6x + 10$

10. \*\*  $7x + 4 = 4x - 23$

11. \*\*  $5(3x - 2) + 4 = 10x + 29$

12. \*\*  $4(2x + 1) - 2(3x - 2) = x + 9$

13.\*\*  $6x - 3(x - 8) = 4(x - 7) + 6$

14. \*\* Solve and graph:  $2x - 1 \leq 13$

15. \*\* Solve and graph:  $-4x + 6 \leq 2x - 30$

16.\*\* Solve and graph:  $2x - 5 > 1$  or  $3x + 2 \geq 14$

17.\*\* Solve and graph:  $-10 \leq 2x + 3 \leq 5$

18.\*\* Solve:  $|2x - 1| = 13$

19.\*\* Solve & graph:  $|2x - 1| \leq 13$

20.\* Fill in the justifications:

$$2x - 1 = 9$$

$$2x - 1 + 1 = 9 + 1$$

$$2x + 0 = 10$$

$$2x = 10$$

$$x = 5$$

Given

\_\_\_\_\_

Identity for Add

\_\_\_\_\_

21.\*\*\* Parental contact information: Provide phone number, home address, email or text. (CHP)

# Sample Linkages

## Example - Linking Multiplication

$$\begin{array}{r} 32 \\ \times 21 \\ \hline 64 \\ 672 \end{array}$$

$$\begin{array}{r} 3x + 2 \\ \times 2x + 1 \\ \hline 6x^2 + 4x \\ 6x^2 + 7x + 2 \end{array}$$

*By choosing simple straightforward examples, the digits and the coefficients are identical in the partial products and final product making math easier to follow.*

## Scaffolding – using the Distributive Property

$$\begin{array}{r} 3x + 2 \\ \times 2x + 1 \\ \hline 6x^2 + 4x \\ 6x^2 + 7x + 2 \end{array}$$

$$\begin{array}{r} (2x + 1) \underline{(3x + 2)} \\ 6x^2 + \underline{4x + 3x} + 2 \\ 6x^2 + 7x + 2 \end{array}$$

## Example – Linkage Order of Operations – Equations

### Order of Operations

1. Parentheses
2. Exponentials
3. Multiply/Divide
4. Add/Subtract

} from left to right

$$5 + 2 \times 10$$

→ 17  
→ 25

Ex.  $2 + 20 \div 5 \times 2 + 1 = 11$

### Elementary School – Solve -Trial & Error

$$4 \times \square + 3 = 23$$

Algebra – Same problem, written differently

$$4x + 3 = 23 \quad \text{Strategy based on Order of Operations}$$

## Example Linkage Transformations

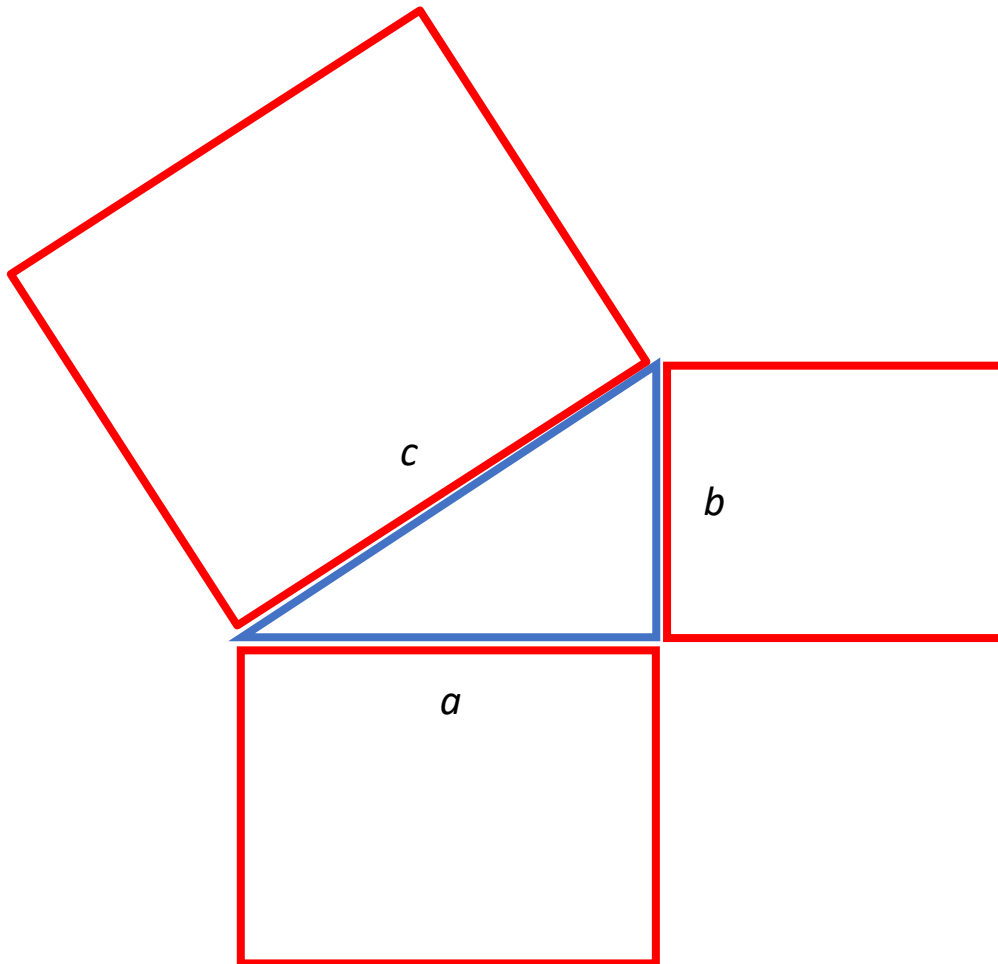
Flips – Slides – Turns

Reflections – Translations – Rotations



**Example – Areas of Squares - Pythagorean Theorem -Distance  
Formula – Equation of a Circle – Trig Identity**

Linkage– formulas the same, just written differently because they are used in different contexts



$$c^2 = a^2 + b^2 \quad \text{Pythagorean Theorem}$$

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance Formula}$$

$$r^2 = (x - h)^2 + (y - k)^2 \quad \text{Equation of a Circle}$$

$$1 = \cos^2 \alpha + \sin^2 \alpha \quad \text{Trig Identity}$$

## Math Content Strategies

### Many Rules in Math Just Don't Make Sense

(Divide by Zero, Zero power, Div of Fractions, Integer Rules)

### Strategies that Make Math Easier to Learn

#### *Common Sense Standard*

#### *My Kid Standard*

Definitions

Linkage

Concept Development

Simple Straight-forward Examples

Patterns

Scaffolding Examples

Compare & Contrast

Think Out Loud

Simpler Problems - Get rid of it

Notation

Models

Notes

Reading

Language Acquisition

Writing

#### *Decision-making*

## Instructional Strategies

**Instructional strategies** are techniques teachers use to help students become independent, **strategic** learners. These **strategies** become learning **strategies** when students independently select the appropriate ones and use them effectively to accomplish tasks or meet goals.

**Lesson Plans**                      **Started Class on Time**    **Check for Understanding**

**Think Pair Share**                      **Students on Task**                      **Students Engaged**

**Good Rapport**                      **Classroom Management**                      **Guided Practice**

**Homework**                                      **Notes**                                      **Questioning**

**Word Walls**                                      **Manipulatives**                      **Cooperative Learning**

**Feedback**                                      **Peer Teaching**                      **Strategic Grouping**

**Do Nows**                                      **Pacing**                                      **Graphic Organizers**

**Direct Instruction**                      **Inquiry Based Learning**                      **Learning Centers**