EOC Math II Remediation Lessons

Nevada Department of Education

Drawing Quadrilaterals

Using Formulas within the Coordinate Plane to Determine Slopes, Distances, and Midpoints

Enduring Understanding

(Do not tell students; they must discover it for themselves.)

Students will use coordinates to prove simple geometric theorems algebraically. Students will explore some properties of quadrilaterals. Students will effectively use formulas within the coordinate plane to determine slopes, distances, and midpoints in order to verify the properties.

Notes

Adapted from Illustrative Mathematics (n.d.). Part 1 and Part 2 are not stand-alone activities and both Part 1 and Part 2 must be completed with ample student discussion before the Synthesize/Apply portion.

Launch (Part 1 of 2)

Draw a quadrilateral *ABCD* with a ruler. Draw your quadrilateral so that no two sides are congruent, no two angles are congruent, and no two sides are parallel.

- Let *P*, *Q*, *R*, and *S* be the midpoints of sides *AB*, *BC*, *CD*, and *DA*, respectively. Use a ruler to locate these points as precisely as you can, and join them to form a new quadrilateral *PQRS*.
- What do you notice about the quadrilateral *PQRS*?

Understand the Problem

- Are there any words you don't understand?
- What are you asked to find?
- Is there enough information to find a solution?
- Can you restate the problem in your own words?
- Or, what information do you need to find?

Develop a Plan

- There are many reasonable ways to solve a problem. With practice students will build skill in choosing efficient strategies.
- Do not validate/invalidate any strategies, but ensure that students have a place to start (even if you know it will not work).
- Do not force your plan/reasoning on students.

Investigate

- Let students engage in a productive struggle.
- Monitor as students work.
- Do not offer feedback.
- Only ask questions.
    - Why did you choose that number?
    - What assumptions did you make?
    - Explain what you are doing here.
    - What does that solution mean?

Questions for Students as they Work

(If you observe _____, then you might ask _____.)

If students do not have a starting point, then ask:

- What do you know about _____ (quadrilaterals, midpoints, congruency, parallel lines, etc.)?
- What does *respectively* mean?

If students experience difficulty labeling, then ask:

- What is a *vertex*?
- What are the rules for labeling diagrams?

If students do not properly place *P, Q, R, S* at midpoints, then ask:

- Why did you choose to place *P*, *Q*, *R*, and *S* there? (Did the student use the ruler appropriately?)
- What is a *midpoint*?
- How did you use the ruler to get the midpoint?

If students experience difficulty finding properties of, or describing, *PQRS*, then ask:

- Tell me one type of quadrilateral. What are its properties?
- Are there any tools you can use to verify the properties?
- Is your inscribed shape a square? How do you know?

Launch (Part 2 of 2)

Draw a quadrilateral *ABCD* on a coordinate plane, where *A, B, C, D* have integer coordinates. Make sure your quadrilateral has no two congruent sides, no two angles are congruent, and no two sides are parallel.

- Let *P, Q, R*, and *S* be the midpoints of sides *AB, BC, CD*, and *DA*, respectively. Locate and label these points as precisely as you can, and join them to form a new quadrilateral, *PQRS*. Recall what you noticed about the quadrilateral *PQRS* in Part 1. Use coordinates to prove the observation.

Understand the Problem

- Are there any words you don't understand?
- What are you asked to find?
- Is there enough information to find a solution?
- Can you restate the problem in your own words?
- Or, what information do you need to find?

Develop a Plan

- There are many reasonable ways to solve a problem. With practice students will build skill in choosing efficient strategies.
- Do not validate/invalidate any strategies, but ensure that students have a place to start (even if you know it will not work).
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Investigate

- Let students engage in a productive struggle.
- Monitor as students work.
- Do not offer feedback.
- Only ask questions.
    - Why did you choose that number?
    - What assumptions did you make?
    - Explain what you are doing here.
    - What does that solution mean?

Questions for Individuals as they Work

(If you observe _____, then you might ask _____.)

If students have a difficult time starting the drawing, then ask:

- What does midpoint mean? What's an idea you have to find the midpoint between two points on a graph?
- What does it mean by *integer coordinates*?
- How do you know your diagram is correctly drawn and labeled?

If students choose to use the ruler to find the midpoint, then ask:

- How do you know that you are making the most precise coordinates or measurements?

If students do not know which formula to use, then ask:

- What property are you trying to prove?
- What formula would help you prove that?
- If you have a formula in mind, what is that formula going to tell you?

If students quickly complete, then ask:

- Is there another set of properties you can prove?

If students have algebraic errors (e.g., calculation errors in determining the slope, but correctly interprets that the slopes are not parallel), then ask:

- What would you expect your answers to be?
- If your outcome isn't what was expected, what are some possible errors you might look for or changes to your approach you might make?

If students misinterpret results (e.g., correct algebra, incorrect interpretation), then ask:

- Which formula did you use and what does it tell you about the diagram?

Debrief

Whole or Small Group Discussion

- Debriefing formats may differ (e.g., whole-class discussion, small-group discussion). It will be beneficial for students to view student work as a gallery walk or similar format.
- Have students sequence multiple representations in an order that moves from less to more mathematical sophistication.
- Allow students to question each other and explain their choices, using mathematical reasoning. If students struggle, model your own questioning strategies.
- Encourage students to notice similarities, differences, and generalizations across strategies.
- Provide purposeful feedback and ask questions.

Questions to Consider

- Looking at all the quadrilaterals created for *PQRS*, what similarities do you notice between them? What differences do you notice?
- What ways can we verify or prove that your observations are true?
- What strategies or tools would you use to prove your observations?

If students struggle answering prompts, then ask:

- What are properties of quadrilaterals (narrow down to square, rectangle, etc., if needed)?
- Do you think we can classify PQRS as something more specific than a quadrilateral?

If you observe *ABCD* as a square, rectangle, parallelogram, or rhombus, then ask:

- Was the diagram constructed using the original guidelines? Why or why not?
- How does that affect the inscribed shape? (This could lead to possible lesson extensions.)

Common Errors

If students' generalizations are too specific (e.g., everyone gets rhombus), then:

- Refer back to the properties versus definition.

If students misclassify the shape *PQRS*, then:

- Review the properties of special quadrilaterals.

If students do not correctly label diagrams, then:

- Students may not understand labeling conventions.

Synthesize and Apply

When students have independently arrived at the Enduring Understanding, engage them in solving these extension problems. Monitor student work and facilitate discussions by asking questions.

Extension Problem 1

- Draw a parallelogram *ABCD* on a coordinate plane, where *A, B, C,* and *D* have integer coordinates.
- Let *P, Q, R*, and *S* be the midpoints of sides *AB, BC, CD,* and *DA,* respectively. Locate and label these points as precisely as you can, and join them to form a new quadrilateral PQRS.
- Classify *ABCD* and *PQRS*.
- Use your coordinates to prove your classification of *ABCD* and *PQRS*.
- Discuss how the classification of *ABCD* effects the classification of PQRS.
- Group students based on classifications of *ABCD* to guide the generalizations for *PQRS.*

Extension Problem 2

- Quadrilateral *ABCD* has coordinates A (2, 6), B (5, -3), C (-4, 0), and D (-5, 7). Classify the type of quadrilateral.

  Solution: Students should prove two pairs of consecutive congruent sides and/or perpendicular diagonals (kite).

Extension Problem 3

- Triangle *XYZ* has coordinates *X* (1, 1), *Y* (-5, 4), and *Z* (-8, -2). Classify the type of triangle based on angles and sides.

  Solution*:* Students should prove that two sides are congruent and two sides are perpendicular (right isosceles triangle).

References

Common Core State Standards Initiative. (2010). *Common core state standards for mathematics.* Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.

Standards

- HSG.GPE.B.4 Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0,2).
- HSG.GPE.B.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
- HSG.CO.C.11 Prove theorems about parallelograms. Theorems include: opposite sides are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

Illustrative Mathematics (n.d.). G-GPE A midpoint miracle. *Illustrative Mathematics.* Retrieved from https://www.illustrativemathematics.org/content-standards/HSG/GPE/B/4/tasks/605, accessed on September 21, 2016, is licensed by Illustrative Mathematics under CC BY4.0

Polya, G. (2014). How to solve it: *A new aspect of mathematical method.* Princeton, NJ: Princeton University Press.

Name_____

Drawing Quadrilaterals (Part 1 of 2)

Using Formulas within the Coordinate Plane to Determine Slopes, Distances, and Midpoints

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- Let *P*, *Q*, *R*, and *S* be the midpoints of sides *AB*, *BC*, *CD*, and *DA*, respectively. Use a ruler to locate these points as precisely as you can, and join them to form a new quadrilateral *PQRS*.
- What do you notice about the quadrilateral *PQRS*?

Name_____

Drawing Quadrilaterals (Part 2 of 2)

Using Formulas within the Coordinate Plane to Determine Slopes, Distances, and Midpoints

Draw a quadrilateral *ABCD* on a coordinate plane, where *A, B, C, D* have integer coordinates. Make sure your quadrilateral has no two congruent sides, no two angles are congruent, and no two sides are parallel.

- Let *P, Q, R*, and *S* be the midpoints of sides *AB, BC, CD*, and *DA*, respectively. Locate and label these points as precisely as you can, and join them to form a new quadrilateral, *PQRS*. Recall what you noticed about the quadrilateral *PQRS* in Part 1. Use coordinates to prove the observation.