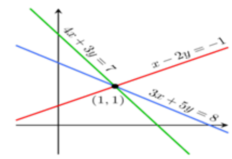



# DDT-cay

## Enduring Understanding

**(Do not tell students; they must discover it for themselves.)**

Students will develop an understanding of the parameters in an exponential function by exploring a rate of decay function to see how different values of the exponent affect the values in the function.

### Standards

HSF.LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.

HSF.LE.A.1.c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

## Launch

### Introduce the Task

DDT is a toxic agricultural chemical that was used in the United States before it was banned in 1972. DDT has a half-life of 15 years. That means it takes 15 years for one half of a quantity of DDT to degrade into a different, harmless chemical. Suppose an environmental scientist in 2015 measured 9g of DDT in a soil sample taken from land where DDT was once heavily used. The scientist modeled the amount of DDT in the soil, *a*, with the function $a(t) = 9(0.5)^t$. She indicated in her notes that *t* represented "time."

a. Find $a(0)$. What might this value represent in this context?

b. Find $a(1)$ and $a(-1)$. What might these values represent in this context?

c. Explain, with more specificity, what you think *t* represents in the function $a(t)$.

### Understand the Problem

- Are there any word(s) you don't understand?
- What is the question or task asking you to answer?
- Is there enough information to find a solution?
- Restate the problem in your own words.
- What additional information do you need to find?

## Develop a Plan

- There are many reasonable ways to solve a problem. With practice, students will build the necessary skills to choose an efficient strategy for the given problem.
- Ensure that students have a place to start and that the task/problem has the ability to be scaffolded.
- Caution should be exercised to not force your plan/reasoning on students.

# Investigate

## Productive Struggle

- Let students engage in productive struggle.
- Monitor as students work.
- Offer positive constructive feedback.
- Ask questions such as…
  - Why did you choose that number?
  - What assumptions did you make?
  - Explain what you are doing here.
  - What does that solution mean?

## Questions for Individuals as they Work

**Students are unable to start the problem …**What do you know from reading the problem? Were you given an equation? What do you need to know to use the equation.

**The student does not understand function notation…**What does the variable *t* represent? What does *a(t)* represent? What does *a(0)* represent? What does *a(1)* represent? What does *a(-1)*represent?

**The student is struggling with a zero and negative exponent...** What is $2^3$? What is $2^2$? What is $2^1$? What is the pattern? So, what do you think $2^0$ is? Does this pattern work for other numbers? So, going a step further, what would $2^{-1}$ equal? Is there another way of writing $2^{-1}$ so that the exponent could be positive?

**The student doesn't know how to explain the meaning of** *a(1)* **or** *a(-1)***….** How much DDT is there initially? How much DDT is there when you calculate *a(1)*? How did the amount of DDT change? How is this change related to the half-life? So, what does *a(1)* represent? What was the value for *a(-1)*? How is this amount related to the initial amount of DDT? How is this amount related to a half-life? What does *a(-1)* represent?

**The student is finished…** Have you verified your solution? How did you verify your solution? Is your solution reasonable in the context of the problem? How do you know? Did you observe any patterns or relationships? What other types of situations could be modeled by this type of a function?

## Sample Solutions

### Possible Correct Response:

Part a:

$$
\begin{aligned}
a(t) &= 9(0.5)^t \\
a(0) &= 9(0.5)^0 \\
a(0) &= 9(1) \\
a(0) &= 9\,grams
\end{aligned}
$$

*a*(0)represents the initial amount of DDT in a soil sample taken in 2015. The initial amount of DDT present is 9 g.

How does this amount relate to the initial amount in the sample? What do you think the 0 in *a*(0) represents? How does the 0 relate to the year in which the sample was taken?

Part b:

$$
\begin{aligned}
a(t) &= 9(0.5)^t & a(t) &= 9(0.5)^t \\
a(1) &= 9(0.5)^1 & a(-1) &= 9(0.5)^{-1} \\
a(1) &= 9(0.5) & a(-1) &= 9(2) \\
a(1) &= 4.5\,grams & a(-1) &= 18\,grams
\end{aligned}
$$

*a(1)* has a value of 4.5 grams. This is one-half the original amount so one half-life has passed. Thus, *a(1)*represents how many grams of DDT there are in a sample taken in the year 2030 =2015 + 15.

*a(-1)*has a value of 18 grams. This is twice the original amount so this represents going back in time one-half life. Thus *a(-1)* represents how many grams of DDT there are in a sample taken in the year 2000 =2015 – 15.

How does the amount 4.5 grams relate to the original amount 9 grams? How does the amount 4.5 grams relate to the concept of half-life? How much time has passed if the sample went from containing 9 g DDT to 4.5 g DDT? In what year would there be 4.5 g DDT in the sample?

How does the amount, 18 grams, compare to the original amount, 9 grams? How does the amount, 9 grams, relate to the concept of half-life? At what point in time would a sample contain 18 grams?

What is the value of $(0.5)^{-1}$? How does this value relate to 0.5? What if we wrote 0.5 as ½ ? What is the relationship between a negative exponent and time in the context of this problem?

Part c: In the function *a(t)*, *t* represents the **number of half-lifes**, or increments of 15 years, that have elapsed before or since the original sample was measured.

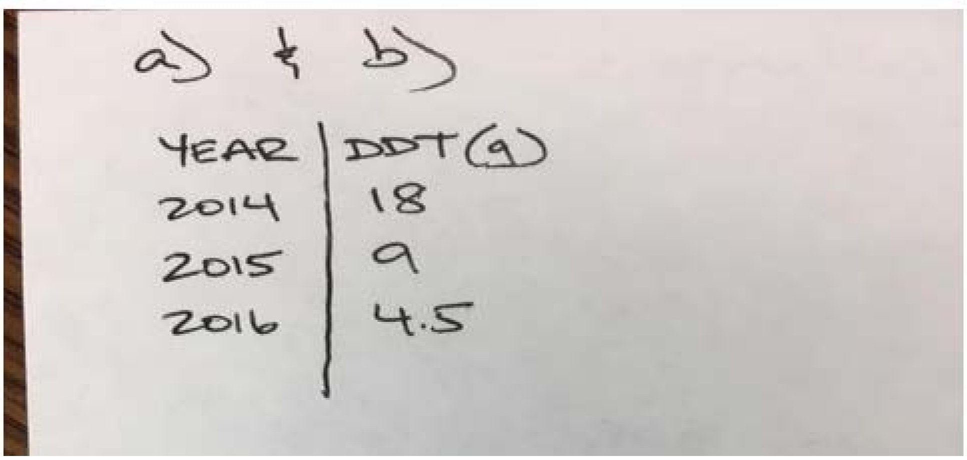
In order for the sample to contain half as much DDT, how many years had to pass? What was the value of *t* in the equation? How does *t* relate to the half-life for DDT? How many years does *t* represent?

## Alternative Strategy

An alternative strategy would be to make a data table. Students would need to include years prior to the initial sample as well as years after the initial sample. Students could also make a column showing the function notation as it relates to the number of 15 year time periods.

| Year | Grams DDT | Function notation |
|---|---|---|
| 1985 | 36 g | $a(-2)$ |
| 2000 | 18 g | $a(-1)$ |
| 2015 | 9 g | $a(0)$ |
| 2030 | 4 g | $a(1)$ |
| 2045 | 2.25 g | $a(2)$ |

## Common Incorrect Response




The student changed the initial year by increments of 1 year, instead of 15 years (half-life).

COMMON INCORRECT RESPONSES

a) $a(0) = 9(0.5)^0$

$a(0) = 0$

THIS MEANS THERE IS NO DDT.

a) $a(0) = 9(0.5)^0$

$a(0) = 1$

THIS MEANS THERE IS ONE GRAM OF DDT.

b) $a(-1) = 9(0.5)^{-1}$

$a(-1) = -9(0.5)$

$a(-1) = -4.5$

THIS MEANS THERE ARE $-4.5$ g OF DDT

b) $a(-1) = 9(0.5)^{-1}$

$a(-1) = 4.5^{-1}$

$a(-1) = \frac{1}{4.5} = 0.2\bar{2}$

THIS MEANS THERE ARE $0.2\bar{2}$ g OF DDT

c) $t$ REPRESENTS THE TIME, IN YEARS

$t = 3$ WOULD BE THREE YEARS, FOR EXAMPLE

# Debrief

## Whole/Large Group Discussion

- Debriefing formats may differ (e.g., whole-class discussion, small-group discussion). It will be beneficial for students to view student work as a gallery walk or similar activity.
- Have students/teacher facilitate the sequence of multiple representations in an order that moves from less to more mathematical sophistication.
- Allow students to question each other and explain their choices, using mathematical reasoning. If students struggle, use questioning strategies.
- Encourage students to notice similarities, differences, and generalizations across strategies.
- Provide constructive feedback and ask clarifying questions for deeper understanding of the process.

**If you see this common error…, it might mean this…**

$a(0) = 9(0.5)^{\circ}$
$a(0) = 0$

The student doesn't understand what a number to the zero power is. The student may also be multiplying by the exponent, 0. Review the zero exponent property. Review the difference between a coefficient, a factor, and an exponent.

$a(0) = 9(0.5)^{\circ}$
$a(0) = 1$

The student may not understand the power of a product property and need help identifying the base-exponent pair. The student may have multiplied (9)(0.5) first and then applied the zero exponent property. Review the order of operations.

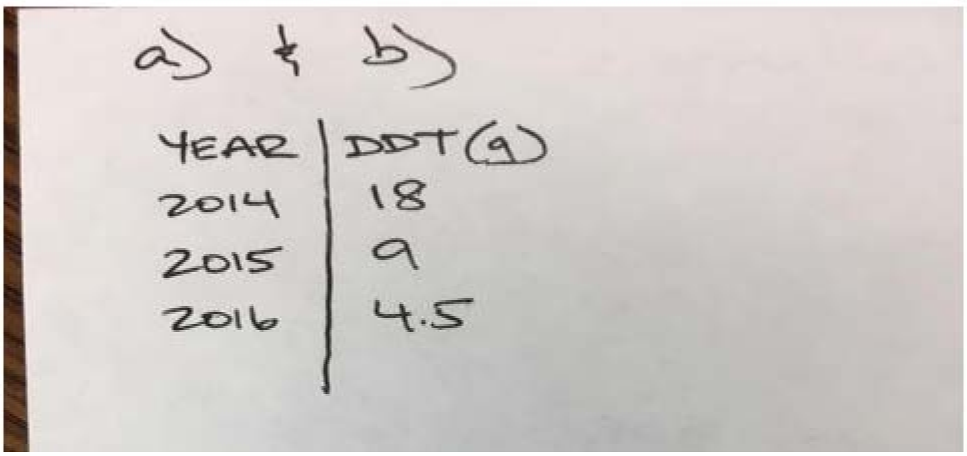
$a(-1) = 9(0.5)^{-1}$
$a(-1) = -9(0.5)$
$a(-1) = -4.5$

The student is changing the exponent to a coefficient. Review the difference between coefficients, exponents, and factors. Also, review the properties of negative exponents.

$$a(-1) = 9(0.5)^{-1}$$
$$a(-1) = (4.5)^{-1}$$
$$a(-1) = \frac{1}{4.5}$$
$$a(-1) = 0.22$$

The student is not applying order of operations. Review order of operations. Review the definitions of exponent, base, coefficient, and factor.

"*t*" is the time in years.
The student is not applying time within the context of the given situation. Review the definition of half-life and review how the amount of the substance is changing in relation to the value of "*t*".




The student is identifying the value of "*t*" as the elapsed time. Review the definition of half-life. Go over a different example with a data table correctly drawn and discuss the change in time with respect to the change in the amount of material.

# Synthesize and Apply

Monitor student work and facilitate discussions by asking questions. When students have independently arrived at the Enduring Understanding, engage them in solving these extension problems. Assess if you have facilitated the discussion in a way that students have arrived at the Enduring Understanding (do not tell them, they will benefit from discovering it for themselves).

## Extension Problem #1

A cancer treatment center uses gold-198 isotope for the treatment of prostate cancer. Gold-198 has a half-life of approximately 2.7 days. The center measured their supply of gold-198 on January 13, 2014 and had 40.0 g. Their supply of gold-198 can be modeled with the function, $g(t) = 40(0.5)^t$, with $g(t)$ being the amount of gold-198 and $t$ represents "time".

a) Find $g(0)$. What does this mean in context?
b) Find $g(1)$ and $g(-1)$. What do these values represent in context?
c) Explain what $t$ truly represent in the function $g(t)$.
d) They need 2.0 grams for a round of treatments on January 23, 2014. Will they have enough gold-198?

## Possible Solution:

*a)* $g(t) = 40\ (0.5)^t$

$g(0) = 40\ (0.5)^0$

$g(0) = 40(1)$

$g(0) = 40$

This represents the amount of gold-198 measured on January 13, 2014.

*b)* $g(1) = 40\ (0.5)^t$

$g(1) = 40\ (0.5)^1$

$g(1) = 20$

$g(1)$represents the amount of gold-198 after one unit of time or one half-life.

$g(t) = 40\ (0.5)^t$

$g(-1) = 40\ (0.5)^{-1}$

$g(-1) = 80$

$g(1)$represents the amount of gold-198 one unit of time prior to January 13, 2014, or one half-life before measuring the amount of gold on January 13, 2014.

c) $t$ represents units of time with each unit being a full half-life of 2.7 days.

d) January 23, 2014 is 12 days after the initial measurement of 40.0 grams. This represents 4.44 half-lifes. Therefore, $t$=4.44.

$g(t) = 40\ (0.5)^t$

$g(4.44) = 40(0.5)^{4.44}$

$g(4.44) = 1.84$

Because 1.84 grams is less than the 2 grams needed, there will not be enough gold-198 for the treatments.

## Extension Problem #2

A person opened and deposited money into a savings account 2 years ago to save for a down payment on a car. The account pays 5% interest annually. The account now has $5,512.50. The formula for interest annually, is

$f(t) = a(1+r)^t$

a) How much money will be in the account one year from now?

b) If the person needs $6,500 for the down payment four years from now, will there be enough money?

c) How much money was initially deposited in the account?

d) Find $f(-4)$. Is this solution possible? Why or why not?

## Possible Solution:

a)

$$f(t) = \$5512.50(1+r)^t$$
$$f(1) = \$5512.50(1+0.05)^1$$
$$f(1) = \$5512.50(1.05)$$
$$f(1) = \$5788.13$$

b)

$$f(4) = 5512.50(1.05)^4$$
$$f(4) = 6700.478$$
$$f(4) = \$6700.48 \quad \text{Need to round to correct cents.}$$

c)

Yes, there will be enough money in the account.

$$f(-2) = 5512.50(1.05)^{-2}$$
$$f(-2) = 5000$$

The initial deposit two years ago was $5,000.00

d)

$$f(-4) = 5512.50(1.05)^{-4}$$
$$f(-4) = 4535.147$$
$$f(-4) = \$4535.15 \quad \text{Remember to use correct decimal places for cents.}$$

This is not possible because the account was not open four years ago.

## Extension Problem #3

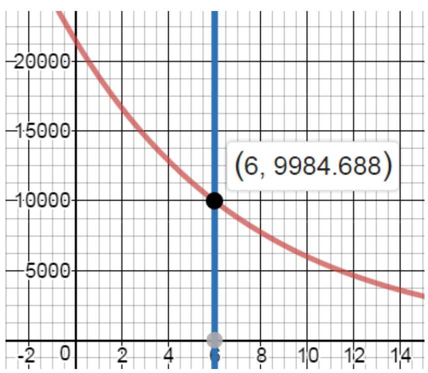
The value of a car is \$21,500. It loses 12% of its value every year.

a) Write a function that represents the value, *y* (in dollars), of the car after *t* years.
b) Graph the function from part (a). Use the graph to estimate the value of the car after 6 years.

## Possible Solution:

a) The initial value is \$21,500 and the rate of decay is 12% or 0.12

$$y = a(1-r)^t \quad \text{Write the exponential decay function.}$$
$$y = 21{,}500(1-0.12)^t \quad \text{Substitute 21,500 for } a \text{ and 0.12 for } r.$$
$$y = 21{,}500(0.88)^t \quad \text{Simplify.}$$

b) 


From the graph, you can see the *y*-value is about \$10,000 when *t* = 6. When you evaluate $y = 21{,}500(0.88)^t$ for *t* = 6, you get \$9,984.69. So, \$10,000 is a reasonable estimation.

## References

Common Core State Standards Initiative. (2010). *Common core state standards for mathematics.* Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.

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**Name______________________________**

# Student Page

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